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Twelve-tone Serialism: Exploring the Works of Anton Webern

A Thesis

Presented to

The Faculty and the Honors Program

Of the University of San Diego

By James Patrick Kinney Music 2015

Introduction

Whenever I tell people I am double majoring in mathematics and music, I usually get one of two responses: either "I've heard those two are very similar" or "Really? Wow, those are total opposites!" The truth is that mathematics and music have much more in common than most people, including me, understand. There have been at least two books written as extensions of lecture notes for university classes about this connection between math and music. One was written by David Wright at Washington University in St. Louis, and he introduces the book by saying "It has been observed that mathematics is the most abstract of the sciences, music the most abstract of the arts" and references both Pythagoras and J.S. Bach as people who noticed a connection between the two long ago.¹ David Benson from the University of Georgia wrote the other one, and he dives right into some questions that the book attempts to answer, such as "How is it that a string under tension can vibrate with a number of different frequencies at the same time?"² I have not looked at questions this complex, but I first became interested in the mathematical composition of music when I composed a piece based off the Fibonacci sequence for a college music class. It was in a music history class covering music from the 19th and 20th centuries that I was first introduced to serialism, and it immediately piqued my interest. My advisor recommended that I read up on twelve-tone serialism particularly, and gave me George Perle's book on the subject to start. He recommended that I look specifically at Anton Webern because he utilized it the most consistently in his works, which would probably be easiest to analyze. I did all of the above and decided to base my thesis on his composition Variations, Op. 27.

¹ David Wright, *Mathematics and Music*, vol. 28 (American Mathematical Soc., 2009), ix.

² David Benson, *Mathematics and Music* (University of Georgia. Department of Mathematics, 2004), ix.

In my thesis, I will provide a background on how twelve-tone serialism came to be, talking about the formation of the Second Viennese School and the origins of atonality. Then I will give an overview of serialism and then dive into the specifics of twelve-tone serialism, with regards to the formation of the row and the placement into music. I will then provide an in-depth analysis of *Variations* and display the conclusions of my research, and finally conclude with ideas for further research.

Background on Atonality

Before looking at the birth of atonality, I will first take some time to describe the end of the Late Romantic period and then see how the atonality of the Post-Great War period arose. The Late Romantic period can hardly be brought up without bringing up Wagner's *Der Ring des Nibelungen*, The Ring Cycle. *The Ring* was Wagner's ideal of *gesamtkunstwerk*, or "total artwork."³ It was his theory that he laid out in a few pamphlets that he believed that *gesamtkunstwerk* was an artistic ideal that eventually "crumbl[ed]…into the individual arts – that of the dance, of music, of poetry, and of the plastic arts."⁴ He believed that it was his task to reunite the arts, and this realization culminated in the twenty-six year process that brought about *Der Ring des Nibelungen*.⁵ Wagner controlled and created every aspect of *The Ring*, from the music to the text to the costumes, and even had his own stage build to stage the production.

The consequence of *The Ring* is that Wagner had developed tonal music to the extent that composers who came after him were left in his shadow and the shadow of *The Ring*. Einstein notes that the only operas that come after Wagner are only worthwhile because they do not

³ Alfred Einstein, *Music in the Romantic Era* (New York: W. W. Norton and Company, Inc., 1947), 235.

⁴ Ibid., 236.

⁵ Ibid., 236-37.

attempt to be heroic and philosophical in the way that Wagner's operas were.⁶ Wagner too, in addition to culminating the development of tonal music, also started the turning point towards atonality with the "emancipation of the dissonance" that started to emerge in his opera *Tristan und Isolde*, with the development of the dissonant Tristan chord.⁷

To look at the birth of atonality, however, we cannot look at Wagner, even though he began the process of emancipating the dissonance. We must turn our attention to Arthur Schoenberg. According to Ethan Haimo, "the birth of atonality was not sudden, did not emerge complete in all of its details...Rather, there was an extended period in which the syntax and idioms characteristic of tonal music gradually disappeared and nontonal procedures began to take their place."⁸ Schoenberg, through the process of composing, gradually arrives at a place that is now considered atonality, although he rejected this term.⁹ This is important, because it is often thought that tonality became so complex (i.e. Wagner) that it was too difficult to "determine contextual definition as described," and as a result, tonal functions were completely abandoned in favor of atonality.¹⁰ While this is essentially what happened, the shift to atonality was much more gradual than this oversimplified idea implies.

To see how atonality arose from tonality, Haimo looks at Schoenberg specifically, because he is undisputedly the leader of the atonal movement and is primarily responsible for bringing about this shift. He looks at one of Schoenberg's essays, *Harmonielehre*, and finds that he has a particular conception of tonality that is not quite dissimilar to many others, and Haimo argues that looking at his theories of tonality can shed light on how his conception of atonality comes about. One thing he notices is that when outlining chord progressions, Schoenberg makes

⁶ Ibid., 257.

⁷ Ibid., 361.

⁸ Ethan Haimo, "Schoenberg and the Origins of Atonality," *Constructive Dissonance: Arnold Schoenberg and the Transformations of Twentieth-Century Culture* (1997): 72.

⁹ Paul Lansky et al., "Atonality," Grove Music Online.

¹⁰ Ibid.

no mention of harmonic progression, but only guidelines for moving from one chord to the next.¹¹ What this means is that he is able to follow his rules of chord progression, but overall, his harmonic progressions aren't necessarily directed towards the tonic, but seem to wander aimlessly. Haimo notices that if Schoenberg restricted his method in a number of ways, such as beginning and ending a phrase on the tonic, relying primarily on diatonic notes, and using only a small number of chord progressions, his chord progressions could start to resemble more traditionally "tonal" music using the idea of figured bass.¹² However, as Haimo notes, Schoenberg rarely places these restrictions on his music, and as a result, his music is not intrinsically directed towards a tonic, even though he utilizes triads and seventh chords and proper voice leading.¹³

Another clue into the idea of atonality from Schoenberg's theory of tonality comes from his view on non-diatonic notes. It is his opinion that one should be free to utilize the notes of any of the church modes at any time, and the consequence of this is that he regards non-diatonic notes as equally viable as diatonic notes, which makes it nearly impossible to distinguish which notes are diatonic, and thus what the tonic is.¹⁴ It is clear to see how this idea results in a disestablishment of tonal hierarchy and paves the way for atonality.

The Second Viennese School

I will now move the discussion to the Second Viennese School and how its constituents arrived at twelve-tone serialism. The Second Viennese School, as Auner outlines, is actually

¹¹ Haimo, "Schoenberg and the Origins of Atonality," 74.
¹² Ibid., 75.
¹³ Ibid., 76.

¹⁴ Ibid., 78.

difficult to define, and it has been defined differently over time.¹⁵ There are various composers whose membership is questionable, but three undisputed members of the school, who I will regard as the sole members, are Arthur Schoenberg, Alban Berg, and Anton Webern, the latter two being Schoenberg's most prominent students. The name, of course, comes from the First Viennese School of the Classical period, namely Haydn, Mozart, and Beethoven. Unlike the original school, however, named as such retroactively, this school was self-identified by its members. Apart from the fact that these composers were all born and raised in Vienna, the name also implies their belief of what they were doing as revolutionizing music as had not been done since the Golden Age of music that was the First Viennese School. Indeed, Perle views their work with serialism and atonality as "the most far-reaching and thoroughgoing revolution the history of music has known since the beginnings of polyphony."¹⁶ The three of them pioneered the world of atonality, and in particular developed the twelve-tone method.

A major part of the journeys of these three composers was learning to live with public hostility and rejection, as it took years for their music to be venerated, or even tolerated. Even as they developed the twelve-tone method, the general public did not understand what they were doing and thus did not appreciate this method and its implications.¹⁷

It is important to note here that although these three composers invented Twelve-tone Serialism, they had no part is inventing the concept of serialism as a whole. Serialism is simply the method of serializing some element of music, meaning that some element of music, whether it be pitch, rhythm, dynamics, etc. is controlled by a series of values that govern its placement

¹⁵ Joseph Auner, "The Second Viennese School as a Historical Concept," *Schoenberg, Berg and Webern: A Companion to the Second Viennese School* (1999): 2.

¹⁶ George Perle, *Serial Composition and Atonality: An Introduction to the Music of Schoenberg, Berg, and Webern,* Sixth ed. (Los Angeles: University of California Press, 1991), xi.

¹⁷ Anne C Shreffler, "Anton Webern," *Schoenberg, Berg, and Webern: A Companion to the Second Viennese School* (1999): 253.

into the composition.¹⁸ Notably, serialism does not automatically imply atonality, but when it is applied to pitch, as it is with twelve-tone serialism, it does end up being a predetermined form of atonality.¹⁹

Anton Webern

Anton Friedrich Wilhelm von Webern was born in Austria in 1883. Shreffler provides a brief overview of him, highlighting that he was "fiercely loyal to his teacher Arnold Schoenberg and to his friends and colleagues" but "openly hostile to nonsympathetic outsiders."²⁰ One important characteristic of his music to highlight is that he "delighted in subtle procedures that can only be revealed through study."²¹ After Schoenberg first introduced a mature twelve-tone method in 1923, Webern worked with it and developed his own style for utilizing this method. At the same time, he returned to writing instrumental music after more than a decade of writing only vocal works.²² At this point I will turn my attention to the twelve-tone method and the specifics of its utilization.

The Twelve-tone Method

I now want to discuss the background of twelve-tone serialism in particular. Schoenberg first described this method in an essay in 1923.²³ Essentially, this method of composition

¹⁸ Paul Griffiths, "Serialism," *Grove Music Online*.

¹⁹ Ibid.

²⁰ Shreffler, "Anton Webern," 251.

²¹ Ibid., 253.

²² Ibid., 285.

²³ Perle, *Serial Composition and Atonality*, 1.

removes any hierarchy of pitch by requiring all twelve pitch-classes of the semitonal scale to hold equal importance and to relate only to each other. This is done by serializing these twelve pitch-classes and putting them in a particular linear ordering. Note that these pitch-classes are the only elements of the music that are serialized – every other attribute, such as rhythm, note value, meter, timbre, and dynamics, is left to the composer's discretion. I will go into the specifics of this technique of composition, starting with defining the row.

The row is simply this linear ordering of the semitonal scale. It must contain all twelve notes and each note appears exactly once. Every note of the semitonal scale is represented by a number, with $C \natural$ as 0, and the ascending scale increasing by 1 until $B \natural$ as 11, as shown:

С	C#	D	D#	Ε	F	F#	G	G#	Α	A#	В
0	1	2	3	4	5	6	7	8	9	10	11

There are various ways to transform the row, and every transformation is "statable upon any degree of the semitonal scale."²⁴ In other words, the composer is able to use any of the transformations of the row when composing. The ways in which the row can be transformed is by transposition, retrograde, inversion, and retrograde-inversion. Note that the transformations of the row always have to contain the numbers from 0 to 11, and this is done by performing addition and subtraction modulo 12. The original form of the row before it has been transformed is called the prime form. To illustrate these transformations, I will use the following as the prime form:

$4 \ 5 \ 1 \ 3 \ 0 \ 2 \ 8 \ 9 \ 10 \ 6 \ 7 \ 11$

This form is denoted as P₄, where the subscript represents the first note of the form.

²⁴ Ibid., 3.

Transposition is simply moving the row up or down some amount, this amount being the transposition number. So if the row is transposed up by 4, the new form is denoted P_8 and is as follows:

8 9 5 7 4 6 0 1 2 10 11 3

The retrograde form of the row is taking one of the transposed forms of the row and expressing it backwards. The retrograde of P_4 is denoted R_4 and is as follows:

 $11 \ 7 \ 6 \ 10 \ 9 \ 8 \ 2 \ 0 \ 3 \ 1 \ 5 \ 4$

The inversion of the row is when the entire row is subtracted from some number, called the sum of complementation. If we take P_4 and take the sum-1 complement (meaning we subtract it from $1 \equiv 13 \mod 12$), we get I₉, where, again, 9 is the first note of the form. This form is as follows:

9 8 0 10 1 11 5 4 3 7 6 2

Finally, we have the retrograde-inversion, which combines elements of the retrograde and inversion by taking one of the inverted rows and expressing it backwards. The retrograde-inversion of I₉ is denoted RI₉ and is as follows:

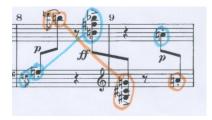
In total, there are forty-eight different forms of the row, because there are twelve prime forms, twelve retrograde forms, twelve inversions, and twelve retrograde-inversions.

I will now discuss how the composer uses the row to compose his piece. First, he decides what the prime form of the row will be. This is where the composer has creative license to compose a row that will determine the theme of the piece. There might be certain interval relationships he wants to highlight, for example. It is important to note that because he has limited control over the notes he can choose, as well as the lack of melody, the rhythmic structure of the theme is extremely important.²⁵

From there, he can place the notes of whatever form he chooses first in any instruments or staves that he chooses, as long as horizontally, the notes are still in the dictated order. For example, the row I have highlighted in the following figure spans both staves of the piano but the notes are still in the correct horizontal order:



In addition, he can choose to employ more than one form simultaneously, resulting in rows that cross over each other, as exampled below, where the orange highlights one form and the blue highlights another:



If there is more than one row being employed, the composer will sometimes put them on separate staves, perhaps crossing them over to a different staff, but that is not a requirement. To make analysis more difficult, multiple rows could completely intertwine. As far as composing chords, the composer can take any sequence of notes in a row and place them in a chord, with vertical ordering being irrelevant. He can even put notes from more than one row in the same

²⁵ Ibid., 61.

chord, as exampled below, where the chord has elements from both the orange form and the blue form:



In-Depth Analysis of Variations

Variations, Op. 27 was composed by Anton von Webern between 1935 and 1936. Comprised of three movements, Webern originally composed the third movement as a piece based upon the variation form where a theme is established, in this case based upon the prime form of the row, and multiple variations of the theme are expressed in succession.²⁶ He then expanded the piece into three movements, but what is perplexing to scholars is that he said in a letter that these two other movements were further variations. Some believe that these movements are in variation form, but more likely, he was expanding the word "variation" to simply mean different types of movements that all derive from the same prime row.²⁷

Before analyzing Variations, I first analyzed the row to find any patterns or properties that could influence the composition. The row that Webern uses in the piece is the same one I used to illustrate the transformations of the row. The prime form, P₄ is, again, as follows:

4 5 1 3 0 2 8 9 10 6 7 11

²⁶ Shreffler, "Anton Webern," 298.
²⁷ Ibid.

What I noticed about the row is that each half of the semitone scale is permuted separately; that is, the first six numbers are a permutation of the numbers 0-5 and the last six are a permutation of the numbers 6-11. It was hard for me to see the implications of this, but what Robert Wason saw was that Webern emphasizes the semitone, which is easily visible here:

Even though the semitones are not always sequential, Webern can still take advantage of them in chords or as points of resolution, in a strange way.²⁸

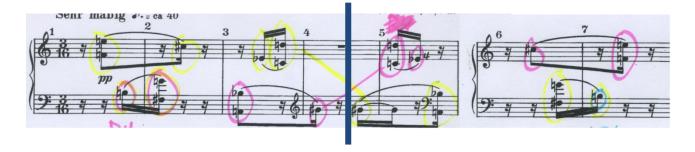
Variation I

The first movement of *Variations* uses one of the classical forms, which is the ternary form, but as an atonal work, this form is really only present in the rhythm, hand placement, and the size of chords, as the two A sections have completely different pitches. However, the phrasing is still heard through the use of *rubato*, particularly with the *ritardando* at the end of each section. The rhythmic structure of the A section is fairly consistent, starting with a pattern of 4 semiquavers and a semiquaver rest, where the first and third semiquaver have a 2-note chord and the second and forth have a single note or vice-versa. This pattern then leads into a continuous stream of semiquavers with alternating amounts of notes in the chords, up to four now, where the single-note semiquavers act as a sort of rest. The B section is much less consistent and now consists of almost entirely demisemiquavers.

The most interesting thing about the structure of the piece, and the most relevant to the serial aspect, is that each phrase is a palindrome. In other words, each phrase can be bisected and

²⁸ Robert W Wason, "Webern's" Variations for Piano", Op. 27: Musical Structure and the Performance Score," *Integral* (1987): 75.

the halves are exact mirror images of each other in terms of pitch and rhythm. What I discovered after finding all of the expressions of the row in the piece is that Webern is able to achieve this by pairing some form of the row with its retrograde, thus creating the phrase. The nature of doing this is that the first half of the original form is identical to the second half of its retrograde and vice-versa. He expresses each form in the phrase in a different hand, and then in the middle of the phrase, swaps the rows so that they switch staves. This way, the first half of the original form and the second half of its retrograde are in one hand and the first half of the retrograde and the second half of the original form are in the other, thus completing the palindrome. An example of this is illustrated below, where the solid line bisects the palindrome:



That being said, however, he does not stick 100% to this formula. In measure 16, for instance, there are two chords in the middle of the phrase that are not the same, but if omitted, the rest of the phrase still forms a palindrome. It is harder to hear that this is a palindrome, though, because one note that is part of the palindrome is in the same chord as three other notes that are not part of the palindrome, so the symmetry is skewed.

Another interesting element of this movement is the way that he separates notes of a chord into the two hands. In most of the piece, whenever there is a chord comprised of elements from both forms, the two hands always separate the forms, meaning that all of the notes from a single form are in one hand, like below:



Occasionally there would be notes that were part of both forms, creating a pivot where the palindrome is bisected. However, towards the end, he begins incorporating chords that do not separate the elements of the two forms into the two hands, but rather blurs them so that one hand is playing a chord that has notes from both rows. An example of this is illustrated below. Here the middle note of the chord is from a different form than the two surrounding notes:



Variation II

Although the second movement utilizes the same row as the first, it is structurally very different. Shreffler calls this a "canonic scherzo."²⁹ The canonic aspect, again, is not in terms of melody, but rather rhythm, note value, the size of the chords, and even the presence of grace notes. The piece is set up in dyads, with the one hand imitating the other typically followed by a quaver rest. Something else interesting about these dyads of notes is that they all have a different dynamic marking, really highlighting the delineation of the pairs. Peter Westergaard provides a detailed analysis of this movement, finding that all of the elements of music are interconnected in

²⁹ Shreffler, "Anton Webern," 298.

this piece, such as certain dynamic markings always accompanying three-note chords and certain note values always following others.³⁰

However, what I am interested in is how the forms of the row influence the piece, and what became obvious after I found all of the forms was that, like the first movement, he composes pairs of forms together, but this time, instead of pairing a form with its retrograde, he pairs it with its inverse with a sum of complementation of 6 (or 18). The result of this is that the sum of every dyad of notes is equal to 6 or 18. Particularly interesting about this is that the inverse of A^{\U034} and D^{\u035} are themselves, because A^{\U035} is 9, which is half of 18, and D^{\u035} is 3, which is half of 6. The consequence of this is that Webern uses A^{\U035} as a pivot point for each phrase, because he always expresses the dyad of them as staccato quavers, making them easily audible:



He does not do the same with $D\Box$, however, because he hides the pair as grace notes, with the exception of measure 11.

Variation III

The third movement of *Variations*, the first one that he wrote, is, again, written in the variation form. This piece was the most simple to analyze because, unlike the first two, he only utilizes one row at a time. At first, I couldn't find a pattern in the theme, which is the first eleven measures, with forms R_8 , RI_{10} , and P_8 other than the fact that P_8 and R_8 both appear, although not

³⁰ Peter Westergaard, "Webern and "Total Organization": An Analysis of the Second Movement of Piano Variations, Op. 27," *Perspectives of New Music* 1, no. 2 (1963).

sequentially. However, I consulted Wason's analysis, and found that instead of using the first form of the *first* movement as the prime form, he uses the first form of the *third* movement as the prime form, which makes sense since Webern composed this movement first.³¹ It is important to note here that this is perfectly valid, since no matter which form we choose to be the prime row, which is an arbitrary assignment, the forty-eight forms of the row are still the same, but now have different denotations. Once I reevaluate my analysis in this way, the pattern becomes clear. Now the forms of the theme are P_3 , I_3 , and R_3 , which combine the horizontal symmetry discussed in the first movement and the vertical symmetry discussed in the second movement.³² Returning the discussion to the implications of the semitone relationships found in the row, one notices that Webern is highlighting this relationship. For instance, in measures 3, 7-8, and 10, he connects notes that are a semitone apart, even though they are not consecutive, which indicates that he wants to emphasize this relationship.³³

The first variation of the theme can be picked out by the structure of the notes. It does not seem to have any obvious rhythmic pattern, similarly to the theme, but the note values are exclusively quarter notes as well as some half notes. There are a lot of two and three note staccato chords interspersed with staccato quarter notes. In addition, this variation is the same length as the theme – about twelve bars. The forms here definitely appear to not have any sort of pattern, but Wason notes that the "row rhythm" has increased significantly; the variation contains 7 forms, whereas the theme has only three. 34

The second variation is distinguished from the first primarily by the motif of a three-note dotted quarter note chord in the right hand that appears five times throughout the variation, which ends at measure 33. What's more, these chords all contain a tritone. As far as the

³¹ Wason, "Webern's" Variations for Piano", Op. 27: Musical Structure and the Performance Score," 73.

³² Ibid. ³³ Ibid., 76.

³⁴ Ibid., 84.

relationship between the forms in this variation, there are a few aspects that stand out. First, they are all retrogrades and retrograde-inversions, which eliminates any sort of horizontal symmetry. In addition, each one has an overlapping note with the adjacent form.

The motif in the third variation is quite obviously the dyads of quavers, which are also semitonal pairs, with one notable exception at measures 35 and 36. There is also a clear horizontal symmetry here, with a palindrome at measures 34-37, as well as measures 38-42. According to Wason, the similarity to the first movement is no accident; Webern created the first movement out of this variation.³⁵

The fourth variation is interesting because it is comprised almost exclusively isolated quarter notes, with some quavers and two-note chords dotted throughout. However, towards the end of the variation, the rhythm picks up and quavers begin to take over in succession while still maintain a quaver rest between them. In addition, the two-note chords become more prevalent, and the variation ends at measure 55 with a fermata. The forms make an interesting pattern. They alternate between a retrograde-inverse and a prime form, and each pair of forms is then transposed up three. The consequence of this is that each form has two overlapping notes with the adjacent one.

The fifth and final variation opens with the very familiar P_3 from the beginning of the theme, and this time moves into a horizontal symmetry with R_3 , although it is not a perfect palindrome. An important difference is that now the E \flat is an octave lower than it had been in the theme, signaling a cadence.³⁶ Webern then transitions into another palindrome in measures 59-63, overlapping the two forms with the entire chord at measure 58. He then brings back R_3 and finishes off with I₃, the third and final form of the theme to show up in this variation. An

³⁵ Ibid., 87. ³⁶ Ibid., 89.

interesting thing I found that must have influenced how he chose his forms is that the chord in measure 58 composed of A^{\natural} , F^{\natural} , and G# that acts as the pivot from P₃ to R₃ remarkably arises again in measure 60 in I₄, which is surprising since these two forms are not directly related to each other, yet the notes in the chord end up sequentially in the two permutations. Similarly, the last chord in measure 58 appears in measure 61 as the pivot from I₄ to RI₄.

Conclusion

When I was first doing this analysis, I was blindly going along and figuring out what the forms were, but what I realized is that the analysis process is so much more involved than simply looking at the forms. The intricate ways that Webern synthesizes the various forms with all of the other aspects of the music is something that I got a hint of reading Wason and Westergaard's analyses, but I realized just how much I don't know and how much higher level music theory goes. Furthermore, I became formally introduced to atonal music, a field with which I was extremely unfamiliar, and, like the majority of the population now and certainly those to whom the Second Viennese School introduced this music, I did not take kindly to it at first, although I wasn't about to start a riot like those seen at the premiers of some of these pieces. This thesis made me appreciate this music so much more, and now when listening to *Variations*, I can pick out a lot of what I have discovered from the score. This music has been called "intellectual music" because it is hard to appreciate without first studying it, but it has gathered a following in the years following its inception, and if it really is as revolutionary as Perle claims, it will hopefully stand the test of time.

Future Analysis

I would now like to briefly discuss further analysis and research that I would liked to have done if time permitted. Firstly, I had planned to analyze another piece of Webern's that was more complex to analyze, and secondly, I would have liked to look more into serialism and other ways to use it.

The other piece I wanted to look at was *Quartet, Op. 22* for clarinet, tenor saxophone, violin, and piano. The reason why I chose this piece was because it does not have as many instruments as his Symphony, for example, but it has enough instruments to pose a significant challenge compared to *Variations*. The more instruments that are used, the vastly more complicated the piece can be to analyze. More instruments mean more possible simultaneous forms, although from what I read, composers rarely used more than two forms at a time. Regardless, it would definitely be more challenging to figure out which notes belong to the same form since they are spread out among four instruments. However, because of time constraints, I was unable to look into this piece, although I would if I had another opportunity to.

The other topic I found very interesting was the idea of serialism and the possibilities of different elements of music to serialize. I was not able to find a lot of research on this topic on first look, but I discovered that there is a concept called total serialism, which is the very idea of taking serialism and applying it to all aspects of music. This was a technique that Messiaen and Boulez both worked on during their careers.³⁷ Although I probably would not be able to study this at the undergraduate level, it is a technique that would be interesting to learn more about in the future.

³⁷ Paul Griffiths, "Messiaen, Olivier," *Grove Music Online*.

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