The Construction of Student Mathematical Identity and its Relationship to Academic Achievement

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THE CONSTRUCTION OF STUDENT MATHEMATICAL IDENTITY AND ITS RELATIONSHIP TO ACADEMIC ACHIEVEMENT

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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DATE: July 9, 2020
Abstract

The California university and state college systems (UC and CSU) are committed to accepting a diverse student body. Although there has been some growth in the percentage of minority students admitted each year, a low number of minority and socioeconomically disadvantaged students meet minimum entrance requirements. For example, in 2018, only 33% of socioeconomically disadvantaged African American students and 39% Hispanic/Latinx students who graduated from California public high schools met minimum UC/CSU requirements (CDE, 2019).

Explanations for ineligibility include the fact that many high school students have not completed the requisite mathematics classes due in part to the inequitable practice of mathematics tracking. Students placed in lower mathematics tracks fail to receive the content they need to gain access to college preparatory math classes. Moreover, students who struggle in math often develop identities of themselves as unable to learn mathematics, beliefs that can have persistent negative effects on their academic outcomes.

In this study, I examined the experiences of ninth-grade students from a majority minority low income high school placed in a lower mathematics track. Unlike their similarly academically placed peers, however, these students were enrolled in a reform-orientated course designed to prepare them to enter the college-going pathway in one academic year. I sought to understand student experiences in the reform course in terms of how their mathematics identities were being constructed in ways that might influence their academic outcomes. To understand the complexities that construct student’s identity and examine that relationship to academic outcomes, a mixed method research design was employed.
Results suggest that there is a relationship between academic outcomes and students’ mathematical identities. This identity is a result of an inextricably interrelated network of influencing factors which include students’ level of confidence in their ability to do math, their grades, teacher/student relationships, and students’ fear of being wrong. Due to the interrelated nature of these factors, results suggest that even addressing one of the factors in this network could impact students’ willingness to engage in class, alter their mathematical identity in positive ways, and ultimately redirect their academic pathway.
I dedicate this work to

my husband Robert and my daughters Jennifer, Amanda and Samantha

for their unwavering support and belief in me.
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CHAPTER 1
INTRODUCTION

The benefits of a college degree remain high despite rising higher education costs (Autor, 2014; Autor, Level, & Murnane, 2003; Goldin & Katz, 2008). Being prepared to do college-level work, or, in some cases, even being admitted to college, can be a challenge for certain students (Musu-Gillette et al., 2016). This is especially relevant as both the California university and state college systems (UC and CSU) are committed to accepting a diverse student population. Although there has been growth in the percentage of minority students admitted to the California higher education system, each year a disproportionate lower number of minority students are enrolled. For example, in 2019, 68% of the UC freshman class were not from underrepresented minority populations1 (University of California, 2019). However, in 2016, 60% of California public high school 12th graders were from historically underrepresented ethnic groups (University of California, 2018). Enrollment challenges in completing requisite coursework in high school can be especially daunting in a field like mathematics where acquiring new knowledge and course progression is, for the most part, facilitated by having acquired prior knowledge in the discipline.

The importance of coursework progression is quite evident in the state of California where both the UC and CSU systems require completion of specific courses to be eligible for admission (Gao, 2016). These courses are referred to as A-G coursework, a series of high school courses students are required to complete for college admission.

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1 Underrepresented minority students are American Indians, African Americans, and Chicano/Latinos.
In 2015, only 41% of California high school graduates completed A-G coursework. The numbers are even more disturbing for the low income Latinx student population with only 32% of these students having met minimum CSU entry standards (Samuels, 2019).

High minority, low income population schools tend to have lower A-G completion rates (Gao, 2016). These are schools in which more than 75% of students are Latinx or African American and/or have more than 75% of students eligible for free or reduced-price lunch, a measure of low income. Further, since 2000 there has been a rise in the number of schools with high minority, low income populations (Gao, 2016; Boschma & Brownstein, 2016). This growth translates into even larger numbers of California students unable to enter the UC and CSU systems. Clearly, the lower A-G completion rates and rise in the number of schools with minority, low income populations could impact and potentially widen the opportunity gap for students in California.

**Assumptions and Access**

The design of the A-G coursework system is based on the assumptions these courses are the body of knowledge students need to show they are capable of college-level material. This systems-driven approach students must go through to qualify for admission into California public higher education institutions disqualifies many underrepresented students. Arguably we need to create a different system that would not impose the kinds of consequences on underrepresented student populations. The need for system-level reform specific to college eligibility is obvious and a need for reform in supporting students’ development of knowledge to enter postsecondary coursework.
Access to A-G courses at high-minority, low-income schools is one explanation for lower A-G completion rates (Raines, 2019). The phenomenon of tracking students by perceived ability, i.e., placing students on various course pathways based on assumptions about their capacity to learn a school subject like mathematics, limits post-secondary options for non-college going tracked students and is another explanation for the lower A-G completion rates for minority and lower income students (Oakes, 1985). Students in lower track\(^2\) courses have decidedly less access to a college preparatory pathway, especially in mathematics (Nasir, 2016). The easy solution would be to not have lower track courses and instead implement differentiated instructional practices, supporting students who have been identified as not ready for college-going coursework. However, a report from the Brown Center revealed tracking practices continue to persist in the United States (Loveless, 2013). Some research suggests high school educators explain placing students in the lower tracks by claiming such placement helps low achieving students catch up to the “regular students.” Unfortunately, research does not substantiate this claim (Mehan, Hubbard, Villanuava, & Lintz, 1996).

Indeed, research has consistently demonstrated placement of students on the lower mathematics track tend to remain on that track and do not catch up to their peers (O’Connor, Lewis, & Mueller 2007; Wheelock, 1992). This is especially troubling when the low mathematics track class is not A-G aligned. Further, teachers assigned to the lower track tend to be less qualified and the curriculum, more often than not, is less rigorous than the college-going track (Nasir, 2016). Additionally, many times school and teacher practices not only limit certain students’ opportunities to learn mathematics; they

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\(^2\) References to course or class tracks and tracking practices in this paper refer to students of the same grade placed into various classes based on assessments that are used to measure mathematics course readiness.
also influence the construction of student views of themselves as learners of mathematics (Tyson, 2011; Spencer, 2006). Students’ view of themselves as learners of mathematics can contribute to mathematical learning (or lack of learning; Berry, 2008; Boaler, 2002; Martin, 2000, 2010; Stinson, 2008) which can, in turn, influence progression through coursework.

**Personal Experiences from the Field**

Ideas about student views of themselves as learners of mathematics and how that impacts academic success has been an area of interest of mine for many years. As a teacher and teacher leader I have firsthand experiences that speak to mathematical beliefs, perceptions and frustrations for both students and teachers that impact student persistence to engage and be doers of mathematics. The following paragraphs capture some of these experiences from the field. These experiences are intended to contribute to the impetus for this research.

**Beliefs**

The consequence of a student possessing a view of themselves as having an inability to learn and be successful in mathematics not only impedes course progression in the secondary setting, but in my experience can also impede course progression in post-secondary settings. Several years ago, I was an adjunct professor at a CSU. The classes I taught were intended for students who did not have a desired score on an entry level mathematics test to enter their field of study. Many of my students were non-traditional college students; students in their last 20’s or 30’s, entering college for the first time or returning after an unsuccessful try at college.
In talking with many of my students I found a common theme. I heard numerous stories about anxiety surrounding mathematics that, to their recollection, started in or around fourth grade. The research of Ginsburg and Asmussen (1988) seems to confirm this notion. According to Ginsburg and Asmuseen by third or fourth grade many students have developed a firm and negative view of themselves as learners. Many of my students felt they were fairly good at mathematics up to fourth grade but as the topics became more challenging, they were left behind in their mathematical understandings and began to develop negative views about mathematics. They felt that this fact along with these experiences hindered their mathematical efficacy into middle school and beyond. A number of them attribute their inability to complete their college degrees to the fact that they couldn’t get “through” the math and as a consequence have negative views about the subject and their ability to be successful. Mathematics can be a gatekeeper to student acceptance into college (Gao, 2016). And although understanding mathematical concepts and possessing skills are important to course success, I believe student mathematical ability-beliefs can also be a gatekeeper to college degree completion. Addressing issues of mathematical ability-beliefs in secondary settings has larger consequences for post-secondary success.

Perceptions

In my years as a mathematics educator, I have come to find perceptions about mathematics as a “hard” subject, not accessible to all, is a prevalent notion. And this perception is used as a way to justify actions of students, teachers, parents, and mathematics departments. One such experience that I share is when I was a middle school mathematics teacher and a department chair. Many times, when I had
conversations with teachers and parents of struggling students, and even with the students themselves, I was astounded by the comments I heard. Typical remarks included:

- Since I teach the low students, I don’t think we should include those students in our data analysis of the math department. They are just always going to struggle in math. (Teacher - mathematics department meeting.)

- Mrs. Trescott, really, I just want my kid to pass math; he struggles you know. Also, I wasn’t very good at math either. (Parent)

- Mrs. T, you know we’re the low math kids, so do we really need to do all this work? (Student - during math class)

These comments indicated a sad reality to me: many people, including the students themselves, did not believe they can learn mathematics, and they were unable to achieve at grade level. This perception was unquestioned; it was just a given. In low-track classroom observations, I found these perceptions were evident in teacher pedagogical practices where expectations and rigor were low. I knew these perceptions needed adjusting, the prevalent idea of “well I was never very good in math” does not just sit with the person who speaks these words, it speaks to a larger perception in our country of the “haves” and “have nots” of mathematics access and it plays out in our classrooms. I believe this perception limits access of groups of the United States population to access equitable education and life options. These perceptions must be addressed if we are serious about equity-based education.

Help!

After four years of asking my middle school principal, I had the opportunity to teach students on the low mathematics track. My year of teaching the low tracked
students was by far the most challenging year I have ever had as a teacher, and the year I grew the most as a mathematics teacher. Every aspect of my pedagogy was challenged. Students did not retain information, students were unmotivated to learn, practices I had always used to get to mathematical rigor did not work with my low tracked students.

I found myself starting to buy into the perceptions that low tracked students can’t learn mathematics. I knew I needed a paradigm shift in my beliefs about my students and in my teaching practices. So, I asked my students for help. Together we worked out a student goal tracing plan that was standards grade-aligned. Students could see their progress and knew exactly where they needed support. I aligned content to those needs. This tracing plan appeared to validate student effort and success which resulted in student motivation and engagement in the learning process.

In speaking with colleagues at other middle schools who also teach low tracked mathematics students I heard the same frustrations I had. One colleague stated, “Many times my students on the low track have had poor or failing grades in math for several years by the time we get them in middle school. My students don’t even want to try anymore.” This frustration was further fueled as schools moved to adopt the state standards (Common Core State Standards), whereby rigor and deep understanding of mathematics is expected of all students and would be testing on state mandated assessments. A colleague stated, “Almost all the students with IEPs (Individual Learning Plans) and ELLs (English Language Learners) go into one class. This makes it very hard, if not impossible, to meet the high expectations of the common core.” One colleague stated, and I agree, “When are we going to actually, really start to address the issues of teaching and learning on the low math track?” In school structures whereby tracking
practices persist, teachers need support to meet the needs of their students. Teacher support must not only address student academic achievement, but also students’ views of themselves as learners of mathematics.

**Statement of Problem**

Because students’ views of themselves as learners of mathematics can contribute to and also, in some cases, inhibit mathematical learning and course progression, research on the construction of students’ views of themselves as learners, referred to as mathematical identity, is burgeoning. The work so far has indicated teaching practices, teacher expectations of student outcomes, peer interactions, and school structures can influence the construction of student mathematical identity in a school setting (Turner, Dominguez, Maldonado & Empson, 2013; Boaler & Staples, 2008; Boston, Dillon, Smith, & Miller, 2017; Aquirre, Mayfied-Ingram, & Martin, 2013; Ashcraft, 2002; Ramirez et al., 2013; Warshauer, 2015; Hiebert & Grouws, 2007; Absolum, 2011; and others). Although the variables identified as contributing to the development of students’ mathematical identities have been varied, one message that comes out loud and clear in virtually all of the research focused on understanding students’ mathematical identity and student outcomes: school context matters. There is a need to continue to research the construction of student mathematical identity in varied school contexts.

One of the most influential studies of how educators and school structures can positively impact students’ mathematical identity and academic outcomes is a study published in 2008 by Boaler and Staples. This was a five-year, longitudinal Standard Mathematics Teaching and Learning Study with three urban high schools in California. One reason the Boaler and Staple’s (2008) study was so influential was that it suggested
specific school factors/features to positively impact students’ mathematical identities and influence academic outcomes. The researchers compared three schools, two of which employed tracking practices and placed freshman mathematics students into either college-going or remedial classes and used “traditional” teaching practices and curriculum. In contrast, the context of the other school in this study, a school they called Railside, placed all freshman students in college-going, de-tracked classes, and used a reform-oriented approach. The ninth-grade students at Railside were academically achieving at significantly lower levels than the two comparative schools at the beginning of the study. Further, Railside demographics were more diverse than the comparative schools with students from a variety of ethnic and cultural backgrounds.

Boaler and Staples reported the students at Railside, the de-tracked school, realized greater academic gains and demonstrated more positive mathematical identities. While the ethnic and socioeconomic composition of Railside were very similar to urban American schools, its practices (during the course of the study) related to mathematics course-taking opportunities differed greatly. The reform orientated approach at the Railside school included both de-tracked mathematics classes, and teaching and learning practices the teachers at the school developed. These teaching practices were grounded in a knowledge base of how mathematics works, the conceptual level of understanding, and on how to teach mathematics for student success of mathematics at this conceptual level of understanding.

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3 The reform orientated approach at the Railside school included both de-tracked mathematics classes, and teaching and learning practices that were developed by the teachers at the school and included an emphasis on conceptual understanding problems and student groupwork.
In contrast to the Railside study, most American schools are not actively de-tracking their mathematics courses (Loveless, 2013). Likewise, unlike the reform curriculum used in the Railside study, teachers are expected to use the district adopted curriculum that may not include a reform approach to teaching and learning (Ladson-Billings, 2008). Further, Railside is a large urban high school, and bused in relatively small numbers of low-income, minority students. However, since 2000, there has been a rise in the number of schools with high minority, low income populations (Gao, 2016; Boschma & Brownstein, 2016). High minority, low income population high schools tend to have lower college-going coursework completion rates (Gao, 2016).

Consequently, there is a need to explore the construction of positive mathematics identities in students and the relationship between these constructions and improved academic achievement in other contexts than the type of context in which Railside was located and structured. Railside was a de-tracked, urban setting that used reform practices for teaching and learning. The proposed study will begin to do this.
CHAPTER 2
LITURATURE REVIEW

If a person does well in something, gets feedback supporting they are in fact doing well, then their attitudes about that thing and their ability to be successful will be positive. Hence, a relationship between successful outcomes and positive ability-beliefs is a concept that is easy to grasp. The converse may not be as obvious. If a person does not do well in something, that might make them want to practice it more and try harder to better the outcome, or it might elicit negative attitudes about their ability, causing them to stop trying or stop the activity altogether. Therefore, knowing the right motivators when a person is not doing well is more complex.

When the relationship about one’s success in activities and one’s beliefs about their ability to engage in the activities is in an educational setting, and in a mathematics classroom, we come to understand it is constructed by various influencing factors. Further, as the classroom is a social setting, these beliefs and dispositions, or a mathematical identity, form as we interact and participate in a community (Martin, 2006). The focus of this literature review is to examine factors in the social setting of a classroom that influence the construction of mathematical identity in relation to academic outcomes.

There are three sections to this review. First, I examined the idea of identity in a community, looking at how interacting with one another impacts the construction of identity. Understanding the construction of student mathematical identity through the theoretical grounding of community is important for this research since the context of this research, low mathematics tracked students, students who have been labeled as
academically not prepared for grade-level content, and reform curriculum, forms the type of learning community investigated in this research. Second, I examined contributing factors in the construction of student mathematical identity in relation to academic outcomes in a classroom, asking in what way are these factors supporting or undermining that construction. Third, I examined the need for mathematics reform in ninth grade mathematics and what is the need and structure of reform to mathematically prepare students for their next math course without the need for remediation in that course.

Learning Community and Identity Construction

The construction of student mathematical identity is complex. As such, it is important to investigate literature to focus my investigation, data collection, and analysis. In this section I explore the literature on ideas that develop a focus and definition of identity used in this study. Further, I explore a theoretical underpinning to ground this study.

Self-identity

Teaching and learning are social events as students and teachers interact in a learning environment. Learning theories that focus on social interactions not only provide a framework for how social interactions affect behavior (Bandura, 1977), but more specifically, how social interactions in a learning community affect student identity. Wenger’s (1998) theory on learning communities posits that we bring our own self-identity into a learning community. As we participate as active members of communities, we construct self-identities in relation to these communities. These participations and identity constructions shape not only who we are and how we interpret what we do, but also what we decide to do (Lave & Wenger, 1991; Wenger, 1998). Martin (2000) added
to this notion of a self-identity stating that an individual does not have a single identity, but instead a collection of identities that together defines how we see ourselves. These identities could be along the lines of race, gender, or, for example, an athlete, among other identities (Martin, 2000). It is important to not make the fallacy of a uniform identity in relation to these various identities (Rouse, 1995).

Téllez, Moschkovich, and Civil (2011) pointed out that a group of students might all self-identify as Latinas, for example, yet come into an educational setting with very different mathematical educational needs. These needs might be influenced by such factors as length of time in the United States, language proficiency in both English and Spanish, consistency or gaps in attendance in school, urban or rural educational settings, and/or socioeconomic status (Téllez, et al., 2011). The point of varied self-identities even in the same ethnic group is important to consider in a school setting in which the majority of students are all classified as Hispanic/Latinx for example, which is the student population of this research. Educators need to be cognizant of the varied self-identities with an ethic group, recognizing non-uniformity of potential student mathematical needs while acknowledging cultural commonalities. It is important to emphasize that cultural communalities can be leveraged to engage students in relevant mathematics (Saifer, Edwards, Ellis, Ko, & Stuczynski, 2010).

Not only do we bring into a learning community a self-identity, as we interact in that community our identities continue to develop in our minds and in the minds of others. Our perceptions of ourselves (i.e., our values, beliefs, desires, motivations, and self-identification) change as we interact and participate in the community. Further,
because culture is socially constructed, culturally responsive teaching practices also align to this theory of learning communities and the influence and development of identity.

**Identity Defined**

Ladson-Billings’ (1994) groundbreaking work in culturally responsive teaching spoke to the need to support student learning by creating social interactions that align to academic expectations, cultural competence, and critical consciousness. Gay (2000) added to the work of Ladson-Billings (1994) by including ideas of validating and affirming student strengths that contribute to building students’ identities. Martin (2000) also added to the body of understanding of meeting student needs through a culturally responsive pedagogy in terms of ways students see themselves as learners of mathematics.

Taking up the notion about our ability-beliefs, the ways others see our abilities, and opportunities to mathematically perform in a classroom, sets learning mathematics as a socialized experience. The construction of these dispositions and beliefs has come to be known as a person’s mathematical identity. Martin (2006) stated,

A mathematical identity encompasses a person’s self-understanding of himself or herself in the context of doing mathematics… It also encompasses how others “construct” us in relation to mathematics. As a result, a mathematics identity is expressed in its narrative form as a *negotiated self*, the result of our own assertions and the sometimes-contested external ascriptions of others. The development of particular kinds of mathematics identities reflects how mathematics socialization experiences are interpreted and internalized to shape
people’s beliefs about mathematics and themselves as doers of mathematics. (p. 206)

Taking up the notion of the construction of identity as a negotiated self being constructed and reconstructed as doers of mathematics in the community, for this study I used Martin’s (2006) definition of mathematical identity defined as “the dispositions and deeply held beliefs that individuals develop, in their overall self-concept, about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives” (p. 206).

Identity is developed in the context of doing mathematics. This definition recognizes one’s ability to participate and perform is a constructor of one’s identity. But students must be afforded opportunities to be doers of mathematics—having entry points to participate and perform. What are those opportunities or factors that allow a student’s entry to participate and perform or not? These are the day-to-day occurrences that occur in the classroom. These are the interactions between students and the teacher. These are the employed teaching practices and structures in the classroom.

**Classroom as Community**

Classrooms are complex, such that many factors, structures and practices can contribute to student learning (Hiebert & Grouws, 2007). As teachers and students interact in the classroom, a limited view of these practices and structures in classroom interactions would occur if we do not take up ideas about identity construction in terms of positionality and power. Individuals possess a positional identity which are the socio-relational structures in our social world (Holland, Lachicotte, Skinner, & Cain, 2001). These structures allow greater or lesser access to activities. Classroom structures and
practices must be in place so as learning is occurring, all students have opportunities to construct positive positional identities, to willingly participate, and perform.

Complexities in the development of identity surfaced in the Martin (2000) study. Martin’s (2000) ethnographic study of African American middle school students suggested there are a variety of factors contributing to African American student outcomes. In this study, Martin (2000) set out to understand the mathematical success and failure among African American youths. He found mixed messages and expectations between community, classrooms, and social groups laid the foundation for a more complete understanding of students’ mathematical beliefs, student understanding to the importance of mathematics, and student outcomes. Martin’s (2000) study recognized influencing factors that are both in and outside classrooms and between peer groups that can affect student identities and academic success.

Students can recognize and respond to peer influences so they can take advantage of positive influences or resist them, both of which contribute to students’ development of their mathematical identity. Martin (2000) suggested some contributing influences to the development of student identities occurs in the classroom, in the interactions of students going against the norms of peers or aligning to them. In a classroom setting, if a student has a nonproductive view of themselves as a learner of mathematics and another student chooses to take on this view, then these interactions can contribute to the continued development of nonproductive identities. This finding contributes to our understanding of the social interactions of students in building mathematical identities and speaks to the importance of the teacher role in navigating these peer interactions.
In Wenger’s (1998) community learning theory, our identities are shaped as we interact and participate in a community. The idea of a learning community whereby peer interactions contribute to the continued development of productive or nonproductive identities is concerning for students on the low mathematics track. Many times, students on the low track have nonproductive identities (Tyson, 2011) and potentially have these identities for many years upon entering high school (Ginsburg & Asmussen, 1988). When students on the low mathematics track interact with one another, there is the potential for the continued development of nonproductive student identities. If a goal in education is to meet the immediate needs of students, preparing them for success in their next mathematics course in school structures that are still tracked, then what are practices that support and shift a community, a classroom of learners on the low track with potentially firm and negative views of themselves as learners of mathematics as they interact in the classroom?

In the Boaler and Staples (2008) study, students shifted their mindset toward mathematics as an enjoyable disciple. Further, students at Railside came to view themselves doers of mathematics and also developed a respect for other students of different cultures, genders, and social classes which are important aspects of a culturally responsive teaching practice (Ladson-Billings, 1994). The findings indicated student relationships to mathematics in terms of interest, authority, agency, and future plans for learning mathematics (course progression) are related. The teaching practices of Railside whereby students presented their math understanding and teachers asked probing questions, provided opportunities for students to advance their dispositions toward mathematics.
Identity is Malleable

The construction of mathematical identity is not linear and in fact has various influencing factors which includes mathematical context. In 2015, Andersson, Valero, and Meaney sought to understand the shifting of students’ mathematical identity in various contexts as it relates to the development of mathematical understanding. In their study, the research participants included 38 upper secondary students who chose a specific course of study that avoided a mathematics-heavy program. During one academic year, students were presented with mathematical tasks in various mathematical contexts. Through surveys, interviews, student blogs and logs, the researchers captured information about students’ perceptions of themselves as learners of mathematics. The researchers found the development of a mathematical identity is not binary; people do not have one identity or another, but instead mathematical identity builds over time and can shift depending on a specific mathematical content (Andersson et al., 2015). Examining teaching practices in the context of shifting math identities, this research found teaching practices that encourage meaningful and engaging discourse positively shifted student mathematical identities (Andersson et al., 2015). Further, less student engagement yielded a less productive mathematical identity. This research found the development of mathematical identities is influenced by discourse in the classroom. This outcome is similar to Boaler and Staples (2008) and Gutstein, Lipman, Hernandez and De los Reyes (1997) in that a key component of building student math identity is facilitated in the classroom; classroom culture and teaching practices directly affects the development of mathematical identity.
Another important implication of the Andersson et al. (2015) study is labeling students with a specific identity is not valid since these identities change as learning occurs. Dweck (2015) warned educators and others about labeling themselves or others into a particular way of thinking or self-identification, specific to possessing one mindset or another. Mindsets are beliefs a person has about the way they learn (Dweck, 2008). Dweck has observed people will self-label or label others as having a specific mindset that is firm and unchanging (Dweck, 2015). Dweck emphasized people do not have one mindset or another, but instead have a mixture of mindsets based on many contributing factors and possessing a mindset is a journey and not fixed.

The idea of cautionary practices in labeling students with one type of identity or mindset is important to consider in equity-based teaching practices in the reform course of this research. The learning community theory tells us identity develops and shifts through interactions with others (Wenger, 1998). Thus, it makes sense that one’s identity also is influenced by interactions with others and that particular interactions can contribute to productive or non-productive development of identities.

Student understanding and awareness of identity as malleable, and in particular it is not fixed, can impact the construction of their identity as well influence academic outcomes. In 2007, Blackwell, Trzesniewski, and Dweck conducted an 8-week workshop with 91 students designed to engage them in understanding how the brain learns to shift students’ views on intelligence. This intervention was conducted with middle school students about a third of the way through the school year. The intervention group showed positive academic outcomes for students who came to realize they had a malleable brain, one that could change and grow as they learned new information (Blackwell et al., 2007).
The results of this intervention are important for the context of my research study since shifting student mindset such that students view themselves as capable doers and performers of mathematics is vital in constructing a positive mathematical identity.

The changing of identity does not only occur in 1 year of school, one teacher, and one school. The malleable aspect of identity occurs even into adulthood. Another outcome from Martin’s (2000) research study was mathematical identity can shift even into adulthood. In this study, not only did Martin (2000) speak to students and conduct classroom observations, he also spoke with many of the parents of the youths in the study. One parent in particular mentioned she had difficulties with mathematics as a child, but as an adult college student who realized math success, she changed her views about herself as a capable learner of mathematics.

Although identity can change as we experience mathematics both as children and as adults, Holland, et. al. (2001) stated, “The long term [identity development], however, happens through day-to-day encounters and is built, again and again, by means of artifacts, or indices of positioning, that newcomers gradually learn to identify and then possibly to identify themselves with” (p. 133). Identity development is therefore both dynamic and enduring and that analyses of how individuals participate is relevant to our understanding of identity formation. Although claims about the impact of a few interactions on one’s mathematical identity may be difficult to substantiate, recurrent experiences participating and being positioned in particular ways have the potential to support long-term identity development (Martin, 2006).

**Theoretical Framing**
The ideas developed in this section point to a theoretical framing of the construction of identity in a community (Wenger, 1998). Specifically, we possess a self-identity that encompasses many types of identities (Lave & Wenger, 1991; Wenger, 1998). We bring this self-identity into the learning community, the classroom, and the interactions in the classroom further shape our identities (Martin, 2000, 2006). This shaping of identities implies that there is not a linear path in this construction, but instead one’s identity is malleable (Andersson et al., 2015; Boaler & Staples, 2008; Martin, 2000). Further, as doers of mathematics, identities are further shifted and shaped by various mathematical contexts in which the community of learners engage (Andersson et al., 2015).

**The Development of a Student’s Mathematical Identity: Four Key Factors**

There are various contributing factors both inside the classroom and in a school structure that influence the construction of mathematical identities. In this section, I outline four of these influencing factors. These factors include: (a) teaching practices, (b) teacher expectations, (c) peer to peer influences, and (d) ability-tracking.

**Teaching Practices**

Researched-informed, equity-based effective teaching practices described in Principles to Actions (Leinwand, 2014) are the “nonnegotiable core that ensures that all students learn mathematics at high levels” (p. 4). These practices are intended to provide teachers with tools that support the learning of mathematics for all student groups. These teaching practices are derived from a body of literature including, but not limited to, interconnected mathematical proficiency strands (Kilpatrick, Swafford, & Findell, 2001), understandings of interactions between teachers and students that inform mathematical
competence (Ball & Forzani, 2011), and the standards for mathematical practice of the Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO], 2010).

The effective teaching practices described in Principles to Actions (2014) are intended to provide access for all groups of learners and to connect teaching practices to the construction of student identity, agency, and competence (NCTM, 2018). The effective teaching practices are:

- establishing mathematical goals to focus learning and structures for participation that build agency (Turner et al., 2013);
- implementing tasks that promote reasoning and problem solving and build students’ efficacy as doers of mathematics (Boaler & Staples, 2008);
- use and connect mathematical representations so as to promote students’ mathematical, cultural, and social resources that validates their experiences and promotes interest in the subject (Boston et al., 2017);
- facilitate meaningful mathematical discourse that develops validation of student thinking and serves to promote confidence in their competence (Boston et al., 2017);
- pose purposeful questions that support the development of positive mathematical identities and agency by situating students as thinkers and achievers of mathematics (Aquirre, et.al., 2013);
- build procedural fluency from conceptual understanding that develops mathematical meaning-making, enhancing student confidence and interest which can lessen mathematical anxiety (Ashcraft, 2002; Ramirez et al., 2013);
• support productive struggle in learning mathematics, providing students time to take ownership in their learning, perseverance, and identity development (Hiebert & Grouws, 2007; Warshauer, 2015); and,

• elicit and use evidence of student thinking not only to inform instruction but also to develop student partnership in the learning process that enhances students’ relationship with the subject and builds agency (Absolum, 2011).

These teaching practices support the characterization of mathematics teaching and learning as a dynamic process that builds new knowledge from prior understandings, and the interactions between teachers and students, and students and students. Thus, the practices are informed by not only research grounded in mathematical teaching and learning, and cognitive science (Bransford, Brown, & Cocking, 2000; Mayer 2002; National Research Council, 2012), but also by learning theories of social interactions.

**Discourse and feedback.** The teaching practices of classroom discourse and feedback to students appear to surface in many studies that focus on student identity (Aguirre, Mayfield-Ingram, & Martin, 2013; Boaler & Staples, 2008; Boston et al., 2017; Gutsytein et al., 1997; Martin, 2006; Warshauer, 2015). Practices such as classroom discourse and questioning techniques that either move learning forward or not. For example, assessing-type questions inform the teacher if students understand an idea or concept or not, but do not necessarily move learning forward (Leinwand, 2014). Whereas advancing type-questions, questions that ask students why something is occurring, for example, allows student to think more deeply about mathematics, put ideas together, and advance student learning (Leinwand, 2014). In low-tracked mathematics classrooms, we
typically see practices that include more assessing than advancing-type questions (Nasir, 2016).

Feedback also either encourages student learning or not. For example, feedback that is simply the conferring of a grade, points, or a mark does not necessarily encourage further learning. Whereas feedback that probes student thinking, asking students to consider ideas that provide student guidance toward successful outcomes, can move learning forward (Black & Wiliam, 1998; Hattie & Timperley, 2007). Tasks can also promote productive student problem solving and can be influenced by teacher expectations of student ability (Boaler, 2015; Cotton & Wiklund, 1989). For example, teachers with high student expectations tend to engage students in problem solving beyond answer-getting, asking students to reason and think and consider other approaches to put ideas together (Cotton & Wiklund, 1989). Feedback that supports students in putting ideas together and building new knowledge is especially important for students on the low track to build grade-aligned mathematical knowledge for next course readiness.

**Teacher Expectations**

Teacher expectations of student outcomes also have an impact on students’ beliefs about themselves as learners and can contribute to student outcomes (Boaler, 2015; Cohen & Garcia, 2014; Shouse, 1996). Although teacher expectations can be formed in a variety of ways and from a variety of sources, most teachers develop academic outcome expectations based on prior information such as past student performance (Cotton & Wiklund, 1989). Negative expectations for student performance are especially troubling
for low tracked students given their assignment to this track suggests a low level of math ability.

As noted, teaching and learning is a social activity, as individuals interact in the classroom students are constructing/adjusting their identities (Martin, 2000). As teaching occurs, specifically student and teacher discourse, teachers may identify specific students as mathematically capable or incapable, which can influence teachers’ expectations of students’ ability to participate in discussion and influence students’ identity.

Heyd-Metzuyanim (2013) conducted a self-reflecting research study with his seventh grade students to understand how the communication interactions between himself, the teacher, and a particular student, Dana, who struggled in mathematics, solidified her self-identity as incompetent in mathematics. Heyd-Metzuyanim (2013) noted that not only did Dana self-identify as “clueless” in mathematics, after an exchange with Dana, he also identified her as “unable to perform the task.” (p. 350). Subsequent interactions with Dana only served to solidify the teacher’s views of Dana as mathematically incompetent. Heyd-Metzuyanim noted,

“there was not much chance for the formation of explorative discourse between Dana and me. My identification of her as ‘clueless’ led me to believe that not much could be gained from asking Dana to express any mathematically rational arguments” (p. 354).

Through reflection, Heyd-Metzuyanim realized that he was examining Dana’s responses through the lens of his own sophisticated mathematical skills in which Dana failed to align, which lowered his expectations of her and prompted less exploratory discourse. Exploratory discussions in mathematics are important for students to develop
mathematics at the conceptual level (Ashcraft, 2002; Ramirez et al., 2013). This research brings forth the importance of examining discourse “from a more neutral viewpoint,” (p. 364), one which is grounded in what students can do to leverage responses to support students to deepen their mathematical understandings. The idea of leveraging what students are capable of doing will be an important teaching practice in a student population of learners who view and are viewed as incapable.

**Peer to Peer Influences**

Peer to peer interactions also contribute to student identity development, which according to Martin (2010) is a key component that influences student ability to learn mathematics. During student interactions, students’ mathematical identity can be influenced by negative and positive identities assigned by other students (Martin, 2000). For example, if the idea that doing poorly in mathematics is seen as positive, this connotation can negatively influence other students’ ideas about learning math and actively inhibit student participation in class. In other words, if a student has a nonproductive view of themselves as a learner of mathematics and another student shares this view, then these interactions can contribute to the continued development of nonproductive identities. This implies that teaching practices that encourage peer to peer interactions and sharing of mathematical understandings can also contribute to identity development, in a productive or nonproductive way.

In the Boaler and Staples (2008) study, students were provided opportunities to justify their thinking about emerging mathematical understandings. Making their thinking explicit was believed to open up the possibility of learning as a class community (Cabana, Shreve, & Woodbury, 2014). Although allowing students to share their thinking
is an important aspect to promoting a productive learning environment (Smith & Sherin, 2019), an awareness of identity development in these interactions is important for a teacher to understand to support learning among all students. Student identities can be monitored and taken into consideration in these interactions.

The idea of solidarity between students in building productive mathematical dispositions and a classroom culture that builds critical thinkers was an important finding in the Gutstein et al. (1997) study. This study incorporated ideas of cognitive science whereby the intent was to build on students’ informal math understanding to develop critical thinkers and connect the students’ culture into the learning experience. The Gutstein et al. (1997) study was a collaboration between the researchers, five bilingual teachers and the school principal who served students in a low-income, predominately Mexican American community. The researchers co-taught and lesson planned with the teachers. Outcomes from this study pointed to the importance of building students’ academic and cultural knowledge as a community of peer learners to develop societal critical thinkers. Personal and cultural shared experiences in a community of learners especially through behavioral prompting and verbal feedback from others are important aspects in forming identities (Holland et al., 2001).

**Classroom environment.** Developing a community of peer learners, whereby students are willing participants in classroom discussions without fear of negative peer influences speaks to the importance of a classroom environment conducive to student participation. During classroom observations, Martin (2000) found students were hesitant to participate if the classroom environment seemed hostile whereby there was a fear that
peer reactions to student correctness or incorrectness might be greeted negatively by other peers.

Sharma, Doyle, Shandil, and Talakia’atu (2011) spoke of the importance of a classroom environment conducive to student willingness to take risks in front of peers. This study was conducted with students in a statistics class to understand how to create a classroom environment where students are willing to make and justify their arguments to the class toward a development of students own critical thinking skills. One finding from this research was students articulated that it was very important to them that their teachers reminded the class of the importance of supportive language when students are working together and sharing thinking both in small group work and in the large classroom setting. One student stated, “Sometimes I don’t have the right question and answer. It is okay to make mistakes. Mr. . . . says we learn from our mistakes” (Sharma, et. al., 2011, p. 300) In other words, it is not enough to provide opportunities for students to participate in classroom discussions, the learning environment must be one in which a student would want to participate and interact with peers, and are willing to make mistakes, an important aspect to learning.

**Ability-tracking**

Many years ago, Oakes (1985) documented the negative effects of tracking practices. The tracking practices Oakes (1985) and others investigated were after the racial integration of schools. Most of the research included large schools with predominately white student populations in which small percentages of African American students were bussed into the school. Researchers found a disproportionate number of students on the low track were African American students.
These low tracks were found to have negative and lasting effects not only on student achievement, but also and ultimately on post-secondary options in life. Students placed on the low tracks were taught at higher rates by low-skilled teachers and with curriculum that was less rigorous than the non-low track (Silver, 1998). The intention of the low track student is to help low achieving students catch up to the regular students (Mehan et al., 1996). However, students placed on the low track tended to remain on that track throughout their schooling (O’Connor, Lewis, & Mueller 2007; Wheelock 1992).

Minorities students placed on the low track were typically low minority, high white student populations. Today, however there is a rise in high minority schools in which tracking practices persist (Loveless, 2013). In these high minority schools, students are placed on tracks by a perceived course readiness or mathematical ability. These tracks, although for the same course, have varied curricula that may be either more or less rigorous (Lucas, 1999; Oakes, 2005).

The analysis by Silver (1998) of the Third International Mathematics and Science Study speaks to the concern that students who are usually placed on the low tracked math class typically have experiences that encompass mostly low-level knowledge and skill such as procedural mathematics. Further, low track instruction has a predominate emphasis that is considered lower in cognitive demand. One finding from a study done by Newmann (1995) and his colleagues of 24 schools, who were amid a school restructure, found the low tracked courses tended to emphasize low-level knowledge, remembering and practice. This low-level instruction puts the student experiences at the lowest levels of Bloom’s Taxonomy of learning.
Interestingly, the positive student academic gains in the Boaler and Staple (2008) study were not in a tracked academic setting and this school has a diverse student population. That is not the case for many mathematics learners today in which tracking practices persist in high minority schools. Negative student beliefs about their ability to learn mathematics often results from placement into the lower mathematics tracks. After all, tracking often conveys messages to students and teachers about smartness and non-smartness of students and students in a lower mathematics track can construct nonproductive views of themselves as learners (Tyson, 2011).

Furthermore, students who are placed in the low mathematics track many times face pedagogy that not only continues to negatively affect their mathematical achievement, but also reinforces the self-image of themselves as struggling learners of mathematics (Oakes, 1982, 1985; Tyson, 2011). Students’ negative belief in their ability as learners can have lasting negative effects on achievement (Martin, 2000).

Since the development of mathematical skills and content are cumulative—mathematics, after all, is a cumulative discipline—students who are behind grade level will consistently be challenged to catch up to their peers and meet academic expectations, particularly if they remain on the lower track. Students who have struggled in mathematics for many years possibly viewing themselves as having an inability to learn mathematics starting in fourth grade, could have firm and negative views of themselves as learners for as much as 5 years upon entering high school (Ginsburg & Asmussen, 1988).

Students who struggle in mathematics class, and placed in the low track, have potentially held firm and negative views of themselves as capable learners of
mathematics for up to five years upon entering high school. The theoretical grounding of this literature review speaks of people bringing a self-identity into a community (Lave & Wenger, 1991; Wenger, 1998). The overwhelming challenge of teaching a low-tracked mathematics class whereby potentially the entire class have identities in the learning community in which they hold firm and negative view of themselves as learners of mathematics is daunting. This illustrates the necessity to be thoughtful and focused on teaching practices that develop productive math identities for students on the low track. The question arises, how do we support student achievement and build productive mathematical student dispositions in low-tracked classrooms whereby students have mathematical identities that include views of an inability to learn mathematics?

**Reform Movement and Research Context**

The question of how to support student achievement and build productive mathematical student identities specific to students who struggle in mathematics has been identified in the mathematics education community as a need. In the following paragraphs I speak to this identified need and the research context and need for specific reform in my research school.

**Need for Reform: High School Focus**

Over several decades there have been various reform initiatives intended to support teaching and learning of mathematics in the United States, including *An Agenda for Actions* (NCTM, 1980), *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), *Principles and Standards for School Mathematics* (NCTM, 2000), and *Focus in High School Mathematics; Reasoning and Sense Making* (NCTM, 2009). The mathematics reform initiatives in the United States have generated long-term
improvement trends at the elementary and intermediate levels (NCES, 2015). However, the high school longitudinal improvement trends have remained relatively constant (NCES, 2015). In fact, National Assessment of Educational Progress (NAEP) in 2015 reported that twelfth-grade students’ overall results in mathematical achievement, and the results specific to race and ethnicity, show no significant change from 2013 to 2015. Additionally, the results of lower achieving students in the 10th and 25th percentiles declined in 2015 (NCES, 2015). This implies that nationally, recent efforts to close the mathematical achievement and opportunity gaps for low achieving and minority high school students have not been successful.

As a result of the long-term flat high school mathematics achievement results, there has been a call to focus reform efforts in mathematics at the high school level. William Bushaw, Executive Director of the National Assessment Governing Board, stated, “We have to redouble our efforts to prepare our students and close opportunity gaps” (Loewus, 2016, p. 2). As a response to the NAEP (2015) results and the longitudinal trend data, Matt Larson, former President of the National Council of Teachers of Mathematics (NCTM), reiterated the need to not only focus on mathematics education to meet the needs of all students, but to specifically focus our attention and efforts on high school mathematics (NCTM, 2018).

The NCTM response to the national mathematics education focus was a 2018 publication outlining a high school mathematics reform initiative framework. This research-based reform framework, Catalyzing Change in High School Mathematics, intends to focus mathematics educators, researchers, and policymakers to embark on discussions in four areas with the following recommendations in each:
1. The Purposes of School Mathematics: Each and every student should learn the essential concepts (as outlined in this reform initiative) to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics (p. 9);

2. Creating Equitable Structures: High school mathematics should discontinue the practice of tracking teachers and the practice of tracking students into qualitatively different or dead-end course pathways (p. 15);

3. Implementing Equitable Instruction: Classroom instruction should be consistent with research-informed and equity-based teaching practices (p. 25); and,

4. Essential Concepts in High School Mathematics: Essential concepts should be incorporated into high school curriculum and represent the most critical content from the domains of number, algebra and functions, statistics and probability, and geometry and measurement which represent the building blocks for foundational mathematics and success in continued study (p. 37).

These focus areas are intended to provide a framework to shift “long-standing beliefs, practices, and policies that are impeding progress” (NCTM, 2018, p. xii). Having a research-based framework is important to understand the work ahead but supporting those who implement change is also vitally important.

Expectations. One of the driving factors of this high school reform initiative, is national and state data that speaks to inequitable student access to mathematics. Examining national and state data implies that there is a measure or expectation of where student mathematical knowledge should be during specific years of learning. Yearly expectations are one of the ideas behind a standards-based mathematics program and
state mandated testing: as student learning progresses over time, testing checks for
growth and the meeting of expectations (CDE, n.d.). As previously mentioned, national
and state data shows that not all students are meeting expectations. If a goal in education,
and specifically this reform, is for students to “expand professional opportunities”
(NCTM, 2018, p. 9) upon completion of high school, then these “expanded
opportunities” must also include opportunities to attend college. There are mathematical
expectations for California students who choose to attend college. College expectations
must be understood if we are to support teachers during this reform initiative and prepare
all students for whatever option they choose. Teachers must align their practices to these
expectations.

California expectations for “well-prepared” students entering college is
documented by the Intersegmental Committee of the Academic Senates (ICAS). The
ICAS is comprised of the Academic Senates of the California Community Colleges, the
California State University, and the University of California. In 2013 ICAS published an
updated “expectations” document in response to the new state standards and was adopted
into California legislation shortly thereafter. The document has two sections, (a) student
and teacher approaches to mathematics teaching and learning, and (b) subject matter.

Section 1 addresses expectations of approaches to teaching and learning
mathematics. For students, these expectations are grounded in productive ways in which
students approach and respond to challenges of new problems or ideas. Students are
expected to have gained these approaches and responses to challenges, referred to as
productive mathematical dispositions, before entering college that support successful
course progression, degree options and degree completion.
These productive student mathematical dispositions include:

- viewing mathematics as making sense;
- confidence and tenacity in approaching new or unfamiliar problems;
- possessing an ability to communicate mathematical understandings;
- accepting responsibility for their own learning;
- holding themselves and others accountable to justify assertions;
- proficiently and confidently using technology to display, explore, manage, and investigate mathematical ideas and conjectures; and,
- holding a perception of mathematics as a unified, interconnected field of study.

These productive student mathematical dispositions of a well-prepared college student align to the “Implementing Equitable Instruction” and the “Purpose of School Mathematics” focus areas of the reform framework (NCTM, 2018). Bringing together both the ICAS college readiness student mathematical dispositions and the reform framework focus areas, implies that teachers are expected to foster a learning environment, using equity-based teaching practices that develop these productive mathematical student dispositions. A challenge to this expectation may be when students enter the learning community already possessing firm and negative views of themselves as learners.

The other section in the ICAS document, Section 2, which speaks to mathematical content students are to engage in while in high school, aligns to the California state standards and the NCTM standards (NCTM, 2000). This implies that the expected mathematical content focus areas for college mathematical readiness, as outlined in
ICAS, are not separate from the California state standards. However, teachers are challenged to teach all of the state standards and to get to the expected levels of rigor or depth of the standards-aligned content that encompass any given course (NCTM, 2018). This challenge is especially poignant for teaching and learning in low-tracked classes in which course work is historically less rigorous than de-tracked classes (Lucas, 1999; Oakes, 2005).

The lack of focus in the high school standards has been hypothesized to be one of the contributing factors that challenge teachers to teach all the standards (Heitin, 2015). The NCTM reform framework addresses the argument of teachers not getting to all the standards through the proposed essential concepts. The essential concepts in the reform framework intend to provide teachers with mathematical focus areas for teachers to provide high-quality, equity-based instruction that develops deep student understanding of mathematics. At the same time that teachers are providing this they are to support the development of productive student dispositions.

A call to focus on high school reform and in particular low achieving and minority students implies a focus on access and equity in mathematics education. Spencer, Battey and Foote (Under Review) examined literature from 10 years of education reform efforts related to how equitable mathematics teaching practices can support student access and learning across student populations. The literature analysis indicates that there is a need to target research of equitable teaching practices in the context of student populations in which mathematics education has not served well. In particular, they insist that research focus on opportunities students have had to engage in mathematics and the opportunities they have had to develop positive mathematics
identities. From the NCES 2015 national data, this implies the need to focus research on those populations that have either shown no growth or a decrease in growth, specifically, low-achieving and/or minority student populations.

**Research Context**

My study is embedded in a larger study funded by the federal government. The larger study is focused on developing and sustaining the capacity of feeder sets of middle and high schools to provide access and prepare all students for the greatest number of postsecondary choices from the widest array of options without the need for remediation. In this larger federal study, a group of professional developers, of which I am a member, developed a reform course intended for ninth-grade students who have not shown mathematics readiness for freshman coursework, specifically Integrated Math 1. The design of the reform course intends to address mathematical unfinished learning from previous courses and prepare students for success in their next mathematics course, Integrated Mathematics 2, without the need for further remediation.

The context of this research is what sets this study apart from other studies that have focused on growth in historically low achieving student populations. In some ways it is an extension of the work of Boaler and Staples (2008) in terms of using a reform curriculum which is standards-grade aligned and develops conceptual understanding of mathematics to understand academic outcomes and the construction of students’ mathematical identity. However, this extension focused on a student population that Boaler and Staples did not study: low-achieving, minority students in a tracked setting in

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4 Unfinished learning is a term used for mathematical concepts or skills that students may have been taught but internalized incorrect understandings or did not yet learn but are needed in order for access to course/grade aligned standards.
which a reform curriculum was instituted. Further, the geographic setting of this research is rural, small town, with agriculture as the main industry. The Boaler and Staple study was in an urban setting. Since context matters when conducting research (Spencer, Battey, & Foote, Under Review) the following paragraphs outline the context of this study, why the school wanted this reform and the struggles that were occurring at the school.

The administrator’s quotations that are incorporated in this section of the paper were captured during an interview in Spring 2020. The two administrators in the interview were the school principal and one of the school’s learning directors who works in the school counseling office. The learning director is a resident of the town and is a graduate of the research high school.

**Tracked setting and the need for reform.** In focusing research on those populations that have shown little or no growth, low-achieving and/or minority populations, many times the schools that these students attend have inequitable structures in place such as tracking. This research study was, in fact, conducted in a tracked school structure which placed ninth-grade students into one of two mathematics courses: the low (non-A-G) course or the college-going (A-G) course. This mathematics-tracked context does not align to the research-based framework outlined in *Catalyzing Change in High School*, which specifically called out the inequities of tracked educational setting and years of scholarship and implored educators to cease that school structure. As such, I am

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5 March 2020 is the time period when most California schools abruptly closed due to the global pandemic, COVID-19, and remained closed for the remainder of the school year. At the time of this interview this school had already been closed nearly four weeks.

6 A-G coursework is a series of high school courses students are required to complete for college admission.
not advocating for a tracked mathematics structure at schools. However, changing school structures and policies on tracking takes time to implement and is “one of the most challenging policy changes to enact” (NCTM, 2018, p. 18). Further, the reality is that the practice of tracking persists (Loveless, 2013).

Since de-tracking is complex and takes time to implement, we need to think about the immediate task of meeting student needs, whether the school has tracking or non-tracking structures. Thus, it behooves educators to support teachers, and researchers to unveil and address, equitable teaching practices in schools where de-tracking has not yet occurred to support students’ content knowledge and productive dispositions, support teachers as they work to meet the needs of all student groups and provide schools with information on potentially de-tracking structures. The persistence of tracked school structures, coupled with students that are behind grade level expectations, raises the question of how do we incorporate equity-based teaching practices, as outlined in NCTM (2014) in our classrooms today, before the laborious structural task of de-tracking occurs to support students to get off the low track and for inequitable school structures to change?

Although this study is situated inside a tracked school structure, the research school Principal has stated that she welcomes the day when all students are placed and successful in a college-going freshman class at her school. The Principal stated that a main reason her school has not de-tracked yet is that currently her teachers do not have the tools/strategies to teach the full set of freshman Integrated Math 1 standards when students do not show mathematical readiness for that content. The principal noted dialogue that has occurred between her and her freshman teachers,
Freshman Teacher: There’s this big gap where the kids don’t get integers\(^7\). Can I completely stop so I can teach them integers?

Principal: Well, you could stop and pause, teach them integers and then move on.

Freshman Teacher: We might need to do this for like 6 or 8 weeks.

Principal: Two months of integers, really? They’re going to tune out just because they’re going to be bored.

The Principal went on to note, “Variables, multi-step equations, whatever the case may be. It comes up with every concept.” The Principal’s concerns about students being stuck remediating mathematics for most of their freshman year is typical of what is seen in a low tracked mathematics course (Newmann, 1995; Silver, 1998).

The Principal noted, however, that her teachers recognize they do not yet have the tools to meet the diverse academic needs of students noting, “They [teachers] want more strategies, they want to build the tool box of first quality instruction.” It is especially difficult for the teachers to teach the conceptual understanding of mathematics, an important aspect of mathematical meaning-making (Ashcraft, 2002; Ramirez et al., 2013). The Principal noted, “Kids need the conceptual piece. And I think it’s hard. I’ve seen even during the [teacher] trainings that it’s hard for teachers to come up with the conceptual piece.”

This struggle of knowing what is needed to meet diverse ability student needs and teacher’s capacity to include these into practice was revealed in a 1998 study by Manouchehri and Goodman. This ethnographic study was conducted with 66 middle school teachers across twelve districts for 2 years, wanting to understand the connection

\(^7\) In standards-aligned curriculum, concepts about integers are typically taught in 6\(^{th}\) grade in California.
between what teachers knew about mathematical content, pedagogical practices and learning theories, and their struggles to implement such practices. A main finding from this study was that teacher’s own lack of conceptual understanding of mathematics got in the way of them being able to teach students conceptual understanding of mathematics (Manouchehri & Goodman, 1998). As the Principal of this reform school noted, her teachers struggle with the conceptual understanding piece in their teaching practice.

These ideas of teachers needing tools and strategies to meet the diverse abilities of their students and the need to be better equipped to teach the conceptual understanding of mathematics are not only reasons for the school to not yet de-track, but are also two of the reasons the Principal stated for the need of this reform course. An additional reason for the need of a reform course is that students at this school are not yet academically successful in mathematics and hold attitudes that they are not good at mathematics. The Principal noted that, “At the end of our first semester with ninth graders, half of them are off track for graduation. And usually it's their Math class that's doing it.” This puts students in ninth grade already behind to meet the A-G coursework requirements for entrance into California’s public higher education system. The Principal’s concern about successful completion of courses coincides with the disturbing 2018 numbers of low income Latinx student populations in which only 32% of these students met minimum CSU entry standards (Samuels, 2019); low income and majority minority student population is the demographics of the reform school. (The reform school demographics are detailed in the Methodology section of this paper.) Thus, students not successfully completing courses is a valid concern for the research school Principal.
The Principal emphasized the other aspect of student success is “getting the kids over the hump of you can do math . . . [there is a] fear of math;” in other words, the beliefs that students hold about their ability to be successful in mathematics. Since academic success is related to student ability-beliefs, it is important that both are addressed in a reform course (Blackwell et al., 2007). The Principal summarized the need for the reform course in this way,

The big thing in our job is to make sure that any kid who wants to go to college can and is successful. So that’s why [we wanted the reform course]. The majority of kids were failing math. They were going in and failing Math 1 [freshman math], moving to Math 2 now they’re failing Math 2 [sophomore math], now they’re failing Math 3 [junior math], you’re looking at a kid their senior year who now have three math classes in their schedule or doing credit recovery because we’re trying to get them out of here. We’re not fixing the problem. We’re not fixing any of it. So the [reform] class was put in place to solve the problem of kids either not being comfortable, not having confidence, not having the skills to get them caught up and then move them comfortably into an A-G class.

**Rural agricultural settings.** Although the context of my research is in a rural agricultural setting, I am not studying rural education in and of itself. However, there is a lack of educational research in rural settings (Arnold, Newman, Gaddy, & Dean, 2005), so it is important not to ignore aspects of the rural setting in this research. The rural setting for this research sits in the central valley area of California, which accounts for nearly a third of California’s population and is the fastest growing population region in the state (Sewarengin & Ramakrishnan, 2019). The California central valley is the
breadbasket of the United States due to its large agriculture industry. The region is geographically bound by the coastal mountain ranges to the west and the Sierra Nevada mountain range to the East. The central valley has seen a population growth over the past decade mostly due to rising birth rates and the large migration of residence from other parts of California in search of affordable housing and jobs (Lillis, 2019). The residents of the central valley are ethnically diverse and the largest minority group (32%) is Latinx (PPIC, 2006). High poverty and lower levels of education are the norm compared to the rest of California (PPIC, 2006).

Rural and small towns make up the majority of the communities in the central valley. One point McShane and Smarick (2019) made is the importance of conducting research through a non-deficient lens since there are many things that are working in rural communities, such as more social unity, stronger beliefs in community wellbeing and stronger community support to one another in a rural community (McShane & Smark, 2019).

The principal suggested the idea of a strong community regarding the school graduation ceremony that would now not be taking place due to the global pandemic, COVID-19, and subsequent school closure. The Principal noted graduation is a very big deal not only for the seniors in high school but for the entire town since everyone comes together to celebrate the students’ accomplishment. The impact of the community not being able to come together for graduation this year is a source of stress for the school and community. School is much more important to the day-to-day life of rural communities than in other settings and as such educational researchers need to
understand this otherwise it could create obstacles in reform initiatives (Arnold, et. al., 2005).

Another point surfaced in the administration interview is a student’s transition to a large university setting from their rural high school. The principal noted they have many capable students in their school who are accepted to attend the large University of California (UC) schools. However, once on a university campus student quickly become overwhelmed with the size and lack of support, finding themselves back in their small town before the end of the first semester. Most students at the high school not only have parents who did not attend college, but who also do not speak English, which the Principal noted as barriers to successful transition to college. The Principal stated,

What happens is our kids are just as smart and as strong academically as any other school. We have a trend of kids getting accepted to these big UCs and they don’t survive the semester or they only survive a semester and they come back home and go to [local community colleges].

Many researchers and practitioners consider first generation and rural college students an at-risk population for college-degree completion (Schultz, 2004). The Principal’s sentiment seems to coincide with a 2004 study in which researchers interviewed six Latinx students from a rural agriculture community in their first year in a large university setting. Results from this study seemed to indicate there were numerous challenges for these students such the academic rigor, school structures and expectations of self-direction of college students. Additionally, student lack of experiences in large towns or campuses also was a transitional challenge (Schultz, 2004).

Conclusion
This research sits in the theoretical framing of the construction of identity in a community (Wenger, 1998). Specifically, we possess a self-identity that encompasses many types of identities (Lave & Wenger, 1991; Wenger, 1998). We bring this self-identity into the learning community, the classroom, and the interactions in the classroom further shape our identities (Martin, 2000, 2006). This shaping of identities implies there is not a linear path in this construction, but instead one’s identity is malleable (Andersson et al., 2015; Boaler & Staples, 2008; Martin, 2000). Further, as doers of mathematics, identities are shifted and shaped by various mathematical contexts in which the community of learners engage (Andersson et al., 2015).

As students and teachers engage in a community of learners, teaching practices that encourage and promote productive classroom discourse and provide student with targeted feedback matter in the construction of identity (Aguirre, et. al., 2013; Boaler & Staples, 2008; Boston et al., 2017; Gutsein et al., 1997; Martin, 2006; Warshauer, 2015). Further, a classroom environment that promotes mathematical risk taking creates opportunities for students to willingly engage in problem solving (Martin, 2000; Sharma, et. al., 2011). Additionally, classroom practices and tasks that promote student access to rigorous mathematics are vitally important for course-level expectations, success in mathematics and the construction of productive mathematical identities (Ashcraft, 2002; Ramirez et al., 2013; Heyd-Metzuyanim (2013).

Although the research is focused on what occurs in classrooms engaged in a reform course, understandings about the rural community is important to consider in the construction of identity (Martin, 2000). This is especially true for high schools in rural communities since what occurs in a rural high school is important to the day-to-day life
of a community (Arnold, et. al., 2005). In the next chapter, I will discuss the methodology used to understand student mathematical identity construction and academic success in the specific context of a rural school of ninth-grade students, placed in low-tracked, reform mathematics classes.
CHAPTER 3

RESEARCH DESIGN, METHODOLOGY AND LIMITATIONS

The purpose of this study was to understand the construction of student identity in relation to academic outcomes of students in a rural, ninth-grade, low-tracked reform mathematics course specifically designed to prepare students to enter a college-going course pathway. The idea of a low-tracked reform mathematics course is unique since typically low-tracked mathematics courses are skills-based and do not use reform practices (Nasir, 2016). The overarching research question was: How do students’ experiences in a high school low-tracked non-A-G mathematics reform course, specifically designed to remediate and prepare students to enter a college-going track, influence the construction of their mathematics identity and impact their academic outcomes? The following research questions guided this study:

1. How are students’ beliefs about their mathematics ability being constructed in the reform class?

2. What factors are supporting or undermining positive mathematics identity and how is math identity related to academic progress in a reform course?

Research Design

Mixed Methods

Both quantitative and qualitative data was needed to develop an in-depth understanding of the complexities of students’ sense of their mathematics ability and its relationship to academic outcomes in the context of this reform course. A mixed method research design was chosen for this study, since the procedure of collecting, and
analyzing and mixing both qualitative and quantitative data provided a greater depth of analysis to help answer the research questions (Creswell, 2002).

Student mathematics identity construction includes complex belief systems of both teachers and students. As Aguirre, et. al. (2013) pointed out, a students’ mathematics identity includes a belief system that crosses many contexts of their lives. Similarly, teacher expectations influence students’ perceptions of their ability and are constructed through many modalities and contexts (Martin, 2000). Thus, the construction of student identity and its relation to academic outcome is complex and includes information that is outwardly observable, seen, heard, or written, and information that is not, but exists in non-verbalized thoughts.

I examined factors that supported or undermined student progress and the construction of their mathematics identity. I conducted classroom observations, interviews with administrators, teachers and students and administered a survey to students to understand their attitudes and perceptions about their ability to do math and how their beliefs are related to academic outcomes.

Surveys and interviews allowed me to access teacher and student voice and add depth of understanding and context. As Patton has explained, interviews and conversations add context to complex phenomenon (Patton, 2015).

Case Study

The overarching research question of this study focused on student experiences in the reform course, along with the apparent impact of this course on both their mathematical identities and their mathematical achievement. The reform course functions as a case, and, therefore, this work is considered a mixed-methods case study. A case
study design was chosen to focus attention on what can be learned from an in-depth look at a single case (Schram, 2006), in this case, the reform course, is bound by time (one academic year) and place (one school site; Creswell, 1998).

**Research Methodology**

**Research Site and Sampling Procedures**

Due to the positionality of the researcher on the professional development team and the fact this school is the only school using the reform course, this study employed both a convenience and purposeful sampling (Neuman, 2011). Although convenience sampling offers low credibility (Glense, 2011), this study will serve as a launch point and initiator of conversations for further investigation with a much larger and more representative sampling of the population.

The research site is a rural, small town California high school with a population of approximately 858 students. The student population is approximately 89% Hispanic, 8% White, non-Hispanic, and 3% African American. The school is considered a high-poverty school, with approximately 80% of the student population identified as eligible for free or reduced-price lunch. The percent of students who did not complete A-G coursework during the 2018-2019 academic year are as follows: African American not Hispanic 77%; Hispanic or Latino 62%; and white 57%. The percentage rates of the research site coincide with state results which shows high minority and low-income schools tend to have lower A-G completion rates (Gao, 2016).

**Students and teacher.** The research site has 10 freshman mathematics classes which are split into two tracks; college-going (A-G aligned) and non-college-going (non-A-G aligned). Three of the non-college-going freshman classes were assigned to the
reform curriculum, and are therefore the three classes included in this study. One teacher teaches all three of these non-college-going reform classes. Data was collected from the reform course teacher and from the reform classes and students.

**Reform teacher background.** The reform teacher is a first-year teacher who recently graduated from college with a mathematics degree. She has yet to complete a teacher training program. She was raised and currently lives in the small town of this study and was a student in the same high school. She is Hispanic and fluent in English and Spanish. Her background is similar to most of her students in that her parents are not fluent in English and did not attend college. Comparable to the school administrator I spoke with and the need to support the small-town community, the reform teacher said, “I knew I wanted to come back and help my community just because I know that growing up I had those teachers who were just nag and didn't actually attempt to make a connection with us or have a relationship with their students.”

The school principal purposefully chose this teacher to teach the reform course. When the reform teacher was a student at the school, the principal was her learning director. The Principal stated that, “I knew her very well and that she was strong in math.”

**Mathematics course placement.** Student placement onto freshman mathematics tracks was determined based on the results of a mathematics diagnostic assessment given to the students during the spring of their 8th grade year\(^8\), the students’ eighth grade second semester grades, and their eighth-grade California Assessment of Student Performance and Progress (CAASPP) scores. Using these assessment outcomes and the school’s

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\(^8\) The diagnostic assessment students took was a high school readiness test published by the UC/CSU Mathematics Diagnostic Testing Program.
placement criteria, 122 (54%) of entering high school students were placed in the college-going track (A-G) classes and 105 (46%) of the incoming freshman population students were placed in non-college-going track (non-A-G) classes.

Amongst the 105 non-college track students, 36 students were identified as nearly-ready for the college track; for this study, these students are referred to as the “nearly-ready” students. The 36 nearly-ready students were placed into two classes. Both nearly-ready student classes and one of the other non-college-going classes (14 students) are using the reform curriculum, and constitute the student groups that will be the focus in this study. Therefore, there are three classes in this study and the total number of students was 50.

**Data Collection Procedures**

Data collection for this study occurred according to the timeline in Figure 1. The types of data collected provided information into the two guiding research questions of this study which are both focused on the construction of math identity and its relationship to academic progress: (1) How are students’ beliefs about their mathematics ability being constructed in the reform class? and (2) What factors are supporting or undermining positive mathematics identity and how is math identity related to academic progress in a reform course?

Quantitative pre and post data from student mathematical assessments was used to understand changes in student mathematical outcomes. A student mathematical attitudes and perceptions survey was used to understand the relationship, if any, between student outcomes and student perceptions and attitudes about mathematics. Qualitative data was collected and used to understand teacher expectations, teaching practices, and peer
influences on the construction of student identity. These data sources included teacher interviews, classroom observations, and student focus group interviews. Additionally, school administrators were interviewed to understand the school need for the reform course and the context of the community. The data collection procedures for both the quantitative and qualitative data are described in detail below.

Figure 1.
Data Collection Timeline

Collecting quantitative data. The UC/CSU Mathematics Diagnostic Testing Program (MDTP) assessments was used in this study to understand student growth and next course readiness. Mathematics Diagnostic Testing Program assessments provide understandings into student math holes or gaps and student next course readiness and are a valid measure when used in this way (Huang, Snipes, & Finkelstein, 2014). Teachers administered the MDTP Algebra/Integrated Math 1 Readiness Test in the first month of school and again at the end of the first semester. The comparison was used to identify academic growth.

Student math attitudes and perceptions survey. The math attitudes and perceptions survey used for this study is a 30 question, 5-point agree and disagree Likert survey. This survey is intended to understand student attitudes and perceptions about mathematics. The survey questions align to the researched ideas of productive math
attitudes and perceptions of the MAPS validation study conducted with college freshman students which distinguishes students’ perceptions of mathematics in authentic education settings (Code, Merchant, Maciejewski, Thomas, & Lo, 2016). There are seven mathematical disposition categories in this survey, when taken together, provide understanding about student beliefs and perceptions about mathematics. The categories for the survey (Code, Merchant, Maciejewski, Thomas, & Lo, 2016) are,

- **Growth Mindset**: “This category rates students’ belief about whether mathematical ability is innate or can be develop” (p. 921);
- **Math applicability in the real world**: “This category is intended to quantify a student’s ability to recognize connections between mathematics and other contexts” (p. 923);
- **Confidence**: The intention of this category is to understand a “person’s perceived ability to successfully engage in mathematical tasks” (p. 920);
- **Interest in engaging in math**: The intention of this category is to understand “a student’s active willingness to engage in mathematical situations” (p. 922);
- **Persistence**: The intention of this category is to understand “how students approach solving a non-routine mathematical problem” (i.e., one where they can ‘get stuck’; p. 921);
- **Sense Making**: “This category is intended to quantify students’ perspectives on the nature of their personal mathematical knowledge” (p. 923) and whether it aligns to simply answer-getting or mathematical; and,
- **Answers to math problems**: “This category characterizes students’ views on the nature of solutions to mathematics problems. Students may view answers in
mathematics as being either right or wrong and the solutions supporting these answers as having a certain degree of rigidity” (p. 923).

For this study, the survey questions were modified to be grade and age appropriate for high school ninth-grade students. Each survey question is either a productive or non-productive way of perceiving mathematics, which contribute to understanding students’ attitudes and beliefs (Code, et. al., 2016).

This survey was administered to students after students have received their semester 1 grades. Administering the math attitudes and perceptions survey after students knew their semester grade is a purposeful design of this research. Since research shows student attitudes and perceptions can be influenced by academic outcomes (Martin, 2010; Boaler, 2015), it was important to capture data about students’ attitudes and perceptions after learning of their most-recent outcomes, their semester 1 grades.

**Collecting qualitative data.** Since teacher expectations of student outcomes has an impact on students’ beliefs about themselves as learners and can contribute to student outcomes (Boaler, 2015; Cohen & Garcia, 2014; Shouse, 1996), pre and post semi-structured teacher interviews was conducted with the intention to understand the teacher expectations of student strengths and challenges related to their beliefs and attitudes, and their academic ability. The first interview was conducted at the start of the study and the second interview near the end to understand changes in teacher attitudes and perceptions in relation to student outcomes.

The five grounding questions in the list below was used to drive the focus and direction of the interview:

1. What is your math learning journey?
2. What are your current understandings about your students’ attitudes toward learning math this year?

3. What are some bright spots you currently perceive in your student attitudes about learning math?

4. What do you think might be some challenges in your students’ willingness to try difficult math tasks?

5. In what ways, if any, do you think academic outcomes are related to students’ attitudes regarding their perceived ability in math?

Question 1 relates to the teacher’s math learning journey. A teacher’s journey, understanding what brought the teacher to the teaching profession, to the field of mathematics teaching and in particular this school, can influence their mindset and expectations toward students (Rattan, Good, & Dweck, 2012) and thereby student construction of mathematical identity (Martin, 2000). Questions 2–5 provided an understanding into teacher perceptions of student beliefs and attitudes about mathematics learning and perceived factors that may be supporting or undermining student outcomes.

**Classroom observations.** There were five days of classroom observations in all three of the research classes. Observations were an important source of data in this study since I was able to understand and capture the context in which the students and teacher interacted which were essential elements to gain a holistic perspective (Patton, 2015). Observations also allowed for an examination of interactions between teacher and students and how beliefs about math identity are shaped. Observations were captured through notetaking using an observation guide prompting particular attention to teacher
and student exchanges through dialog, teacher instructional practices, and student engagement in mathematical content and activities.

**Student focus group.** Six students participated in a half hour focus group interview. Students volunteered to participate. However, under direction of the school Learning Director, a purposeful sampling of students was selected. There were three students who earned an A at the end of the first semester, two who earned a B or C, and one who earned a D or F. Academic outcome purposeful sampling was done because of the connection between academic outcomes and mathematical identity. I wanted to make sure I have a mix of student academic success.

This was a semi-structured student focus group interview to understand students’ beliefs about themselves as mathematics learners. This group interview was conducted near the end of the research study after all other data are collected and analyzed (except the administrators interview), and emergent themes have been identified. Students were questioned about their mathematics identities. Information gleaned from the student focus group, with students interacting in the present, was important to this study since it added strength in my understanding from other qualitative data in this study (Neuman, 2011) and in particular increase meaning and validity in my findings since perspectives are formed in the context of a social group (Patton, 2015).

**Data Analysis Procedures**

The data analysis process and procedures were organized around the guiding research questions of this study according to Figure 2. Details of the analyses are further described below.
Figure 2.

**Data Analysis Process**

<table>
<thead>
<tr>
<th>Question #1: How are students’ beliefs about their mathematics ability being constructed in the reform class?</th>
<th>Question #2: What factors are supporting or undermining positive mathematics identity and how is mathematics identity related to academic progress in the reform course?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Attitudes and Perceptions Survey</strong></td>
<td><strong>Teacher Interviews (Pre/Post)</strong></td>
</tr>
<tr>
<td><strong>Survey Purpose:</strong> Understand productive and unproductive student beliefs</td>
<td><strong>Observations Purpose:</strong> Understand teaching practices and student interactions that might influence mathematics identity and academic progress</td>
</tr>
</tbody>
</table>
| **Analysis Procedure:** 1. Data was coded by productive vs. unproductive beliefs 2. Data was also categorized into the seven mathematics perceptions and attitudes categories from the survey | **Analysis Procedure:** 1. Data was categorized into  
  - Teacher questioning and feedback,  
  - Language in student interactions, and  
  2. Teacher questioning and feedback was further analyzed by questions and feedback that either advances or assesses student understanding  
  3. Student language while interacting in peers was analyzed as deficient or encouraging |
| **Relational Analysis:** Overall outcomes were examined in relation to student outcomes, looking for a relationship or connection | **Relational Analysis:** Overall outcomes were examined in relation to the student attitudes and perceptions survey, looking for a relationship or connection |
| **Triangulation of data sources to inform overall research question:** 1. Student focus group interview 2. Classroom observations | **Triangulation of data sources to inform overall research question:** 1. Student focus group interview 1. Student attitudes and perceptions survey 2. Teacher interviews |
| **Academic Outcomes (Pre/Post)** | **Classroom Observations** |
| **Assessment Purpose:** Understand student mathematics growth | **Observations Purpose:** Understand teaching practices and student interactions that might influence mathematics identity and academic progress |
| **Analysis Procedure:** Percent change was calculated for each students’ percent correct score | **Analysis Procedure:** 1. Data was categorized into  
  - Teacher questioning and feedback,  
  - Language in student interactions, and  
  2. Teacher questioning and feedback was further analyzed by questions and feedback that either advances or assesses student understanding  
  3. Student language while interacting in peers was analyzed as deficient or encouraging |
| **Relational Analysis:** Overall outcomes were examined in relation to the student attitudes and perceptions survey, looking for a relationship or connection | **Relational Analysis:** Overall outcomes were examined in relation to the student attitudes and perceptions survey, looking for a relationship or connection |
| **Triangulation of data sources to inform overall research question:** 1. Student focus group interview 2. Classroom observations | **Triangulation of data sources to inform overall research question:** 1. Student focus group interview 1. Student attitudes and perceptions survey 2. Teacher interviews |
Analysis Procedures for Question #1: How are students’ beliefs about their mathematics ability being constructed in the reform class? As noted in Figure 2, there are two data sources used to provide information and understanding for how student beliefs about their mathematics ability are being constructed: the student attitudes and perceptions surveys, and the pre and post student academic outcomes assessments. Each data source was analyzed by itself and then in relation to one another to understand the correlation or connection, if any, between student beliefs in their mathematics ability and their academic outcome.

First, at the individual student level, mathematics pre and post assessment data was examined to understand changes in academic growth over the course of the semester. Second, class level post-assessment data was examined alongside class-level mathematics attitudes and perceptions survey data to examine any relationships. Note: This analysis could not be done at the student level since student names are not on the surveys. However, the survey data are identified by class therefore a class-level analysis was done.

The class level analysis was used to compare academic outcomes with perceptions and attitudes between the three. The goal of the analysis was to understand if there exists a difference between the three classes. As discussed in the student sampling section of this paper, two of the reform classes have students identified as “nearly ready” for the college-going track and one class has students that were identified as not ready. Therefore, a comparative analysis between the two “nearly ready” and the one “not ready” student classes was conducted to understand beliefs and perceptions across these student populations.
Analysis Procedures for Question #2: What factors are supporting or undermining positive mathematics identity and academic progress in the reform course? As noted in Figure 2, there were two data sources used to understand factors that support or undermine the construction of student identity and academic progress: teacher interviews and classroom observations. Additionally, student focus group data was captured which also informed this guiding research question. However, focus group information relates to the overall research question, and as such, is described in the “overall results” section below. The analysis of data obtained through the teacher interviews provided understanding of teacher expectations of student outcomes and perceptions the teacher believes may contribute to the construction of student mathematics identity. Additionally, since these interviews were both near the start of the study and the end, analysis of interview data looked for changes in teacher expectations that may substantiate or add depth of understanding into the observed teaching practices from the classroom observations.

As noted in the informing literature section of this paper, teacher expectations of student outcomes can influence teaching practices (Boaler, 2015; Cotton & Wikeland, 1989), specifically feedback to students and classroom questioning techniques, which I also captured in the classroom observations. Conducted analysis of the teaching practices specific to feedback to students and teacher questioning techniques, as well as other teaching practices that emerges during classroom observations which appeared to influence the construction of student identity and academic outcomes. Additionally, in the Boaler and Staples (2008) study, the teaching practice of student groupwork appeared to influence student outcomes and create an atmosphere of peer to peer support of
productive identity development. Since student interactions can be influenced by negative and positive identities of students (Martin, 2000), analyzing student group interactions in terms of the language students use that could influence student identity construction was also analyzed.

**Overall analysis.** The understandings gleaned from the data analysis directed from the guiding questions was used both as the focus and direction for the student focus group interviews (this is the last piece of student and teacher data captured in this study), and a triangulation of data to inform the main research question: How do students’ experiences in a high school low-tracked non-A-G mathematics reform course, specifically designed to remediate and prepare students to enter a college-going track, influence the construction of their mathematics identity and impact their academic outcomes? Triangulation of data is important to add accuracy in reporting and deepen understandings (Neuman, 2011).

The purpose of the student focus group interview was to deepen emerging understandings of student experiences and add student voice. Information obtained from the student focus group was coded into student experiences that appear positively or negatively impact the construction of mathematics identity and academic outcomes. Understandings that emerge from the student focus group was examined in relation to the other emerged themes and outcomes in this study toward a deep understanding of student experiences and in particular the construction and relationship of student mathematics identity and academic outcomes.
Research Limitations

I identify four limitations to this study. First, the participants in this case study were from one school, with one teacher and her students. Thus, there is no way to generalize the findings in the traditional scientific sense. Instead, the idea of generalizability was looked at through the psychological lens of expanding, enriching and understanding the social constructs of teachers in the field (Donmoyer, 1990).

A second limitation was my subjectivity as the researcher. As previously mentioned, I am part of the professional development team of the larger study in which this research resides. My role in the professional development team was capturing, analyzing and progress monitoring of student academic outcomes and teacher pedagogical development. My need to step in and try to immediately problem solve with the teacher during the semi-structured interview portion of the study, instead of sitting back and just gathering data was difficult. Thus, I used a procedure that enacted an awareness of this subjectivity and the potential impact that it might have on the study in general and on the data analysis in particular. Frequent monitoring or taming my subjectivity (Eisner & Peshkin, 1990) was necessary to manage this subjectively and create a narrative rooting in the stories of the research participants.

A third limitation of this study was timing. The reform course used in this study is designed to prepare students for rigorous A-G coursework. Although the focus of this study was understanding the construction of student mathematics identity in the reform course in relation to academic outcomes, extending this study over a longer period of time would have provided a deeper understanding of the long-term aspects of the construction of identity for the students. Further, extending this study over a one or two
academic years would add understanding in determining how prepared and successful students are for subsequent and rigorous college-going coursework as they progress through high school.

A fourth limitation of this study was with the data capturing procedures. Classrooms and other social settings are rich in information (Patton, 2015). In this study I wanted to see and understand how peers interacted, what was occurring during group work, how the teacher interacted with student groups, with individual students and discussions in the whole class setting. This was a difficult task to do on my own. Having another researcher in the classroom with me in another area of the classroom, listening to and capturing other student and teacher conversations would have helped gather more information and would have added even more validity to my findings. Additionally, observations are subjective; each person brings into an observation their own interests, biases, and background (Patton, 2015) even with a valid observation tool and protocol. Thus, having another set of eyes and ears would have also added an additional layer of validity to the findings.

An additional aspect of the data capturing limitation was the administration of the student math attitudes and perceptions survey. This survey was only administered once, near the end of the study. To understand growth in student mathematical attitudes and perceptions, a pre- and post-survey could have been conducted. Further, it would have been useful to have student identifying information on the survey (student name or ID) to conduct a correlation analysis between student attitude and perceptions from the survey and their academic outcome. This would have added to the validity and strength in the connection between mathematical disposition and academic outcome.
CHAPTER 4

RESULTS

“One of the things that I wanted to do was to help them [her students]

because the majority of them don’t believe they can actually learn.”

Reform Teacher

I report the results of the study in two sections. In section one, the results are
quantitative, describing academic success and student mathematical attitudes and
perceptions. Descriptive statistics and correlation results are presented in this section. The
second section is qualitative in nature. Descriptions and quotes highlight factors that
support or undermine the construction of positive student identities and academic
success.

As previously indicated, there are three classes of reform courses. All of the
students in this study are ninth graders, whom the school placed into two categories of
classes: nearly mathematically ready and not mathematically ready for Integrated Math 1
(IM1); students from both categories participated in this reform course. Students in the
Class Period 5 are students who were initially identified as not mathematically ready for
IM1 and students in Class Periods 1 and 2 were all identified as nearly mathematically
ready. All three classes received the same reform support and materials from a
professional development team. The two distinct student populations of this study,
students nearly mathematical ready and not mathematically ready, prompted a
comparative analysis between class periods. Additionally, an overall analysis was done to
understand what is occurring across all reform classes. This overall analysis and a class period comparative analysis are discussed in the forthcoming paragraphs for both the quantitative and qualitative results.

**Quantitative Results**

The quantitative results section is organized into two main sections: academic achievement and student mathematical attitudes and perceptions. In each of these sections, I analyze overall results and results by class.

**Student Academic Achievement**

Student mathematical achievement was captured using the UC/CSU Mathematics Diagnostic Testing Project (MDTP) student readiness test for an Integrated Mathematics 1 (IM1) course. MDTP assessments are designed to expose gaps in mathematical proficiency and to determine readiness for the subsequent course, Integrated Math 2; MDTP assessments are a valid measure when used in this way (Huang et al., 2014).

Students were administered the same MDTP test twice: at the beginning of Semester 1 (pretest) and the end of Semester 1 (posttest). The same student population and number of students, \( n = 45 \), were administered both the pre- and posttests, and the same test items were analyzed. The test has 40 multiple-choice items, categorized by the test developer into eight mathematical topics; 18 of those items were selected for comparison across the different types of students in this research. The chosen 18 items align with the content taught during the fall semester. The mathematical topic areas included in these 18 items are:

- Data Analysis and Probability & Statistics
- Integers
Overall results. Overall results show the majority of students, 67%, demonstrated growth in their percent-correct score between the pre- and posttest. These results indicate, after the first semester, students in the reform classes appeared to have achieved a greater readiness for course content aligned to an Integrated Math 1 course than they did at the start of the semester.

The mean percent correct score is higher on the posttest, 43.3, than the pretest, 32.8, as shown in Table 1. To verify if student growth between the pre- and post-scores was statistically significant or appeared by chance, the mean scores were compared using a two-paired sample correlation since the student populations and assessment items were the same for both the pre- and post- data set. The $p$-value in the last column of Table 1 is the “level of significance” and indicates how likely the observed differences are the result of random chance. A $p$-value of 0.05 means the observed differences are less likely to happen by chance 1 in 20 times. By convention, the significance level of 95% is used. The comparison between the pre-and post-scores indicates $p = 0.002$, less than our significance level of 0.05, which seems to indicate statistical significance. The actual $p$ level implies growth results from the reform course are not likely to have occurred by chance.

Table 1.

**Assessment Results: Mean Percent Correct**

<table>
<thead>
<tr>
<th>Percent Correct Scores: Statistical Significance</th>
<th>Fall (Pretest)</th>
<th>Winter (Posttest)</th>
<th>$t$ (level of significance)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean score Std Deviation n</td>
<td>Mean score Std Deviation n</td>
<td>$(p &lt; 0.05)$</td>
<td>$p = 0.002$</td>
</tr>
</tbody>
</table>
Results by class period. The mean comparative results between the three reform classes are shown in Table 2. The p-value in Class Period 2 (nearly ready students) of $p = 0.02$ is less than our significance level of 0.05, indicating statistically significant differences between pre- and postscores for that class. The other two classes, Class Period 1 (nearly ready students) and Class Period 5 (not ready), do not have p-values less than our significance level; this demonstrates that although there was growth in their mean scores, we cannot with confidence state the growth that occurred was outside of the realm of chance with any confidence.

Because the student data are from an MDTP readiness test for an IM1 course for college-going students, it is important to examine the class means to begin to understand student mathematical course readiness. This is an important consideration since the objective of the reform course is to successfully prepare students to enter a college-going course next year, IM2. Higher posttest mean scores could indicate higher levels of course readiness.

Class Period 5 has a pretest mean score lower than the other two classes. This relatively low score is not surprising since this class has a student population was assessed as having less course readiness at the start of the semester. Although Period 5 showed growth between their pre- and post-mean scores, the growth was not statistically significant. The class posttest score was still lower than the other two classes, 37.2, as shown in Table 2. The only class indicating statistical significance in growth between their pre- and post-MDTP assessment is Class Period 2, with $p = 0.02$. Class Period 2 is one of the classes whose student population were identified as nearly mathematically
ready for IM1. Although academic growth was realized in all three class periods the mean percent-correct scores on the posttest was less than 50% indicating students would have enough mathematical tools and skills to successfully engage in less than half of the mathematical concepts in IM1 at the end of semester one of their high school freshman year.

Table 2.

Assessment Results by Class Period: Mean Percent Correct

<table>
<thead>
<tr>
<th></th>
<th>Students initially identified as nearly ready for IM1</th>
<th>Students initially identified as not ready for IM1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Class Period 1</td>
<td>Class Period 2</td>
</tr>
<tr>
<td></td>
<td>n</td>
<td>Mean</td>
</tr>
<tr>
<td>Pretest</td>
<td>19</td>
<td>34.8</td>
</tr>
<tr>
<td>Posttest</td>
<td>43.6</td>
<td>17.20</td>
</tr>
<tr>
<td>$t$ (level of significance)</td>
<td>$p = 0.115$</td>
<td></td>
</tr>
</tbody>
</table>

Student Mathematical Attitudes and Perceptions

Students were administered a math attitudes and perceptions survey after receiving their first semester grades. The 30 question, 5-point agree and disagree Likert-type survey consisted of statements that either indicated a productive or nonproductive attitude or perception about mathematics. For some questions, an answer of “disagree” or “strongly disagree” indicated a productive disposition, and for other questions an answer of “agree” or “strongly agree” indicated a productive disposition. The 5-point scale was collapsed into two categories: productive or nonproductive disposition. The answers chosen as “neither agree nor disagree” were placed into the nonproductive belief category. Twenty students took the survey in Class Period 1, 15 students in Class Period
2, and 13 students in Class Period 5, therefore 48 student responses are included in the mathematical dispositions dataset.

**Overall results.** More than half of the student responses, 55%, indicated a productive mathematics disposition, as shown in Table 3. I examined student responses in the seven disposition categories\(^9\) to better understand student beliefs and attitudes in each particular disposition. The productive mathematical disposition categories above the overall average were: sensemaking, 70%; real-world connections, 57%; and growth mindset, 66%. The highest percentage of these was mathematical sensemaking. The sensemaking category results of 70% indicates many students hold a productive disposition about learning mathematics for understanding and suggest such learning is important to them. The growth mindset results reveal that about two thirds, 66%, of students hold the idea that mathematical ability can be developed and is not static. In the category of real-world connections, over half of the students, 57%, believe mathematics applies to everyday life; in other words, they see the usefulness of mathematics in their lives.

On the other hand, the categories with student productive mathematical dispositions of percentages below the overall average were: confidence, 49%; interest, 50%; persistence 47%; and answers to problems, 43%. The answers to problems category was the lowest productive disposition. Results in this category indicate most students in the reform courses at the end of semester see mathematics as a collection of facts and not as a unified field of study. Additionally, students feel less confident, interest, and willingness to persist in solving math problems. These nonproductive views may inhibit

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\(^9\) Mathematical disposition categories of this survey were described in the methodology section of this paper.
students from tackling unfamiliar problems and, overall, make them less willing to engage in mathematics.

Table 3.

*Percentages of Student Productive Disposition Responses*

<table>
<thead>
<tr>
<th>Mathematical Disposition Categories</th>
<th>Productive Mathematical Disposition Class Averages</th>
<th>Overall Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period 1</td>
<td>Period 2</td>
</tr>
<tr>
<td>Growth Mindset</td>
<td>63%</td>
<td>63%</td>
</tr>
<tr>
<td>Real World</td>
<td>54</td>
<td>55</td>
</tr>
<tr>
<td>Confidence</td>
<td>46</td>
<td>63</td>
</tr>
<tr>
<td>Interest</td>
<td>43</td>
<td>60</td>
</tr>
<tr>
<td>Persistence</td>
<td>45</td>
<td>58</td>
</tr>
<tr>
<td>Sensemaking</td>
<td>73</td>
<td>71</td>
</tr>
<tr>
<td>Answers to Problems</td>
<td>41</td>
<td>42</td>
</tr>
<tr>
<td>Average Percent</td>
<td>(53)</td>
<td>(59)</td>
</tr>
</tbody>
</table>

Results by class period. The mathematical disposition data were disaggregated by class period, since the school placed students into two categories of classes, *nearly mathematically ready* (Class Periods 1 and 2) and *not mathematically ready* for IM1 (Class Period 5). Class Period 2 had the highest percentage of productive mathematical dispositions held by students, 59%, as shown in Table 3. This result means students in Class Period 2 hold attitudes and beliefs about mathematics that are more productive in terms of learning mathematics than the students in the other two class periods. Class
Period 5 appears to have the lowest percentage of productive dispositions but only differs from Class Period 1 by one percentage point. Further, both Class Periods 1 and 5 are below the overall average of 55%, and Class Period 2 is above the average.

Comparison of the disposition categories between the class periods reveals Class Period 5 has the largest range, or spread, of productive dispositions with some categories showing a high percentage of productive dispositions and some showing a very low percentage of productive dispositions. For example, Class Period 5 has 73% student productive beliefs about growth mindset, yet only 38% student productive beliefs about persistence and confidence in mathematics, as shown in Table 3. This result means students in Class Period 5 hold nonproductive views in their confidence and persistence to tackle unfamiliar tasks, yet understand mathematical ability can be developed.

The range of productive dispositions for each class period is shown in Table 4. Class Period 2 not only shows an overall mean score that is higher than the other two class periods, but also a smaller range in the student attitudes and beliefs. This result shows, for the most part, students are in more agreement in terms of their attitudes and beliefs about mathematics compared to the other two reform classes. It is also interesting to note the standard deviation for Class Period 2 is the smallest of the three class periods indicating there is less variance in that class in the types of productive mathematical dispositions.

Table 4.

<table>
<thead>
<tr>
<th>Class Period</th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Range Score of Productive Disposition by Attitude or Belief Category</th>
</tr>
</thead>
</table>

*Descriptions of Productive Mathematics Dispositions*
Quantitative Summary

In this summary, I place the quantitative results of the academic outcomes and the math dispositions survey side-by-side to understand if there appears to be a relationship between the two sets of quantitative results. This summary is only the beginning of our understanding of this relationship. Possible influencing factors that support or hinder this relationship were also captured and will be discussed in the next section. Table 5 shows both the academic outcome and the productive disposition results overall and for each class period.

Table 5.

*Academic and Disposition Results Summary*

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>Students initially identified as <em>nearly ready</em> for IM1</th>
<th>Students initially identified as <em>not ready</em> for IM1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Class Period 1</td>
<td>Class Period 2</td>
</tr>
<tr>
<td>Postassessment mean percent-correct score</td>
<td>43.3</td>
<td>43.6</td>
<td>49.1</td>
</tr>
<tr>
<td>Statistical significance in academic growth (95% confidence level)</td>
<td>Yes ($p = 0.002$)</td>
<td>No ($p = 0.115$)</td>
<td>Yes ($p = 0.020$)</td>
</tr>
<tr>
<td>Percentage of student responses that indicate a productive</td>
<td>55%</td>
<td>53%</td>
<td>59%</td>
</tr>
</tbody>
</table>
Although the academic growth was statistically significant overall, the mean percent-correct score on the postassessment was less than 50%, see Table 5. Since the MDTP postassessment used to generate these results helps educators understand student readiness for IM1 and the postassessment included only mathematical concepts students engaged in up to the end of first semester when this test was administered, results indicate students appears to understand less than half of the mathematical concepts in the course they are currently enrolled and are not yet ready, by the MDTP indicator, for the majority of the concepts in IM1. Additionally, results from the mathematical attitudes and perceptions survey indicate about half of the student responses indicate productive depositions in mathematics. It is not clear if these overall results are correlated, although the mean percentages appear to coincide in terms of students with productive dispositions and student success on mathematical problems; the higher the academic outcome mean, the higher the student disposition mean.

The comparison by Class Period of academic success against productive dispositions seems to indicate a relationship. Class Period 2 had the highest post- assessment mean percent-correct score, and the growth was statistically significant; students in Class Period 2 also had a higher percentage of productive dispositions with a narrower range across the dispositional categories than in the other two classes, as shown in Table 5. Conversely, Class Period 5 had the lowest post-assessment mean percent-

<table>
<thead>
<tr>
<th>mathematical disposition</th>
<th>27</th>
<th>32</th>
<th>29</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productive disposition range across categories</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
correct score, with academic growth that is not statistically significant, and the lowest percent of student responses indicating a productive mathematical disposition.

**Qualitative Results**

In this section of my results, I dive deeply into data that examine the opportunities student had to construct a positive student identity and academic success. This evidence was captured from teacher interviews, a student focus group discussion, and numerous classroom observations. The two categories of coded results were: factors in these opportunities that “support the construction of a positive student mathematics identity and/or academic success” or “undermine the construction and/or academic success.”

After the results were coded, certain themes emerged. These themes suggest that although opportunities were provided to students, there is a complex relationship between various factors that appear to influence the construction of positive student mathematical identity and/or academic success, undermine it, or have a variety of influences. Discussed themes are (1) relationship between grades and confidence and student willingness to participate; (2) classroom norms and practices that appear to form an interplay between teacher expectations and student dependence on the teacher; (3) influencing factors other than grades and/or confidence that seem to support or hinder student willingness to participate in classroom tasks and activities; and, (4) the importance of relationships and relatability in the construction of student identity and academic success. This qualitative results section of the paper is organized by those emerging themes.

**Relationship Between Grades and Confidence and Student Participation**

Frequent and varied assessments and assessing strategies are a desired pedagogical practice that can help a teacher understand student progress and needs.
Results from teacher and student interviews and classroom observations appear to show that although the teacher provided students with opportunities to show what they know, results from this study indicate that when students receive grades/marks from assessments or tasks, this influenced student confidence and willingness to participate and as such, there appears to be a relationship between grades, confidence and willingness to participate. I explain this relationship in the following paragraphs.

**Grades: Confidence and ability beliefs.** Students for the focus group and the teacher indicated academic outcomes in terms of grades on assignments or semester grades can have a positive and a negative influence on student confidence and beliefs about their ability to be successful in mathematics. All six students in the focus group indicated grades were important and can impact their ability-beliefs, in other words, how they feel about their ability to be successful in mathematics. Students suggested grades can both support or lower confidence. One student stated, “If you do good on it [mathematics], then you know, you’re good at it [emphasis added]” The student’s comment suggests doing good “on” mathematics, that is having successful grades, influences their belief about whether they feel they are good “at” the subject.

The teacher’s sentiments seemed to support this idea. The teacher stated that many times “if a student gets a bad grade . . . they look at it, and like most students do, crumple it up.” The teacher, however, pushes students to look at their improvement over time instead of a single grade or score on an assignment. The teacher stated she tells students, “Hey, look at what you did on your previous assignment, have you improved?” It is not clear if the teacher explicitly helps students make the connection in specific skills and concepts that the student is showing growth in, or if this teacher’s comment is more
The teacher believes grades on assignments can influence students’ confidence if it is messaged using the kind of “growth mindset” language that focuses on improvement. One student insightfully indicated that when he gets a grade in a semester it influences his confidence and perseverance but when grades are really low, confidence waivers:

Mostly I think grades [influences confidence] because if you start off with a B or C in the class, you know you can do good in that class. But if you’re already starting off the first week and you already have an F – it [grades] can either boost or lower your confidence.

This student’s thoughts about grade achievement indicates confidence is somewhat volatile. Both the teacher and students indicated that if a student enters high school as a freshman already possessing confidence in mathematics they might have developed in middle school or they developed due to another influence outside of school, they will try but as they confront additional challenges, their efforts may diminish. This quote from a student who struggles in mathematics seems to capture the relationship between confidence and persistence in mathematics. The student stated, “Confidence. If you lack confidence and tell yourself you can’t complete an assignment then you’re going to fail, because you feel you can’t do it. So, you’re not going to give your full potential.”

Both the teacher and students indicated the relationship between grades and confidence starts prior to high school. The teacher indicated that at the start of the first semester, most of her incoming students labeled as not ready for freshman mathematics lacked confidence. She stated, however, “The students who appeared to possess
confidence were the ones who were already achieving their goals [obtaining a passing grade].”

**Grades: Motivation to participate and perform.** All six students from the focus group confirmed grades were tremendously important to them and even can impact their high school experience and motivation to participate and succeed in part because grades affect their participation in other “fun” high school activities. One student stated, “It [grades] allows you to do the fun things in high school” and that “fun” can be taken away, impacting students’ willingness to engage in and learn mathematics. The following student quote nicely captures this idea:

For me, I did struggle with math in middle school and now, and honestly . . . it stressed me out because I stopped doing sports. I stopped doing everything else. Just to work on that. And it actually would be nice if I could let the teachers and everyone know so they can let the other kids at least continue to do what they want to do. And at least know their math. Take the stress off of them. Make it easier for them to learn.

The teacher explained that some students don’t really understand how the high school system works and that it is not the same as middle school. The teacher noted, “They’re freshman, they’re getting used to coming into a new school. They don’t see the importance of turning stuff in or they don’t see how it would negatively impact their grade.” Furthermore, the teacher explained there are influencing factors outside of the school setting that might be impacting success and beliefs about students’ willingness to participate in learning mathematics. The teacher indicated that of her 50 freshman students, about 30 students appear to not care about grades. At the same time, she
believes all students can be successful and that some do want to learn. The teacher expressed the dilemma she faces. While she really wants to meet the needs of all her students, she struggles to understand why so many appear to not care:

It’s a drag to see that your students are not performing the way they would like. I tell myself if I put the effort to help all my students – then the ones who actually take little bites [students internalize information the teacher is teaching], come after school and ask me questions, that’ll be enough. But I still have those 30 students who just, it seems like they don’t care. So, it has to be some outside factor that’s going on that I can’t fully comprehend. I do have those students who care about grades and will ask what’s going like ‘why does this happen,’ but I still have those students who are just not catching on, I have about a handful of students in each class [who want to learn].

The teacher’s idea that students “don’t care” about grades runs counter to the comments made by the six students who participated in this study’s focus group. These students expressed that they are motivated and do care about grades. Students even indicated they would like to be given the option to do extra credit to bring their grades up. I asked all the students if they were provided the opportunity to do extra credit would they, and there was a full consensus and resounding “Yes!” I then asked, “and is that because grades matter?” I received another full consensus and resounding “Yes!” I then pushed, “would you say grades are super, super important to you?” Again, I was greeted with a full consensus of “Yes!”

These counter narratives persist with the teacher thinking students don’t care about grades and thus, don’t care about learning math and the students indicating that
grades are “super” important to them. These competing notions between the teacher and students about grades in relation to students’ motivation and willingness to participate and perform were related to student confidence and their beliefs about their mathematical ability. It is not clear which came first; whether success builds confidence or confidence yields success. What is certain is that there appears to be a relationship between grades and its impact on student willingness to participate, ability-beliefs, perceptions about willingness to learn, confidence and ultimately academic outcomes and that there exists a lack of shared understanding between the teacher and her students.

**Classroom Norms and Practices: Teacher and Student Interplay**

In each of my classroom visits, students were provided opportunities to work together with peers on mathematical tasks. Providing students with opportunities to work together is a teaching practices widely used in U.S. classrooms and has the potential to yield student success (César, 2007; Esmonde, 2009; Sung, 2018). Although students were provided opportunities to work together, the majority of interactions were between teacher and student and not between students with students as would typically be the case during group work. The interactions between teacher and student during group work classroom time revealed itself as an interplay co-created by the teacher’s expectations of the students’ willingness and ability to complete mathematical tasks and students’ dependence on the teacher for mathematical support. In the following paragraphs I elaborate on this interplay and its influence on students’ ability and participation in mathematical tasks by examining the classroom norms and practices.

**Group work norms and practices that yield student dependence on the teacher to complete tasks.** Although students were provided with opportunities to work
together, group work did not necessarily yield student participation or success on tasks. In fact, as students worked in groups there appeared to be a student dependence on the teacher to complete tasks. There did not appear to be guidance provided to students about group work structures or how students are to work together (communicate, share strategies, what to do when they are all stuck on a mathematical idea or concept).

When students were asked to work in groups the teacher encouraged them to help each other on the provided mathematical task. During group work in these classes, students sat in clusters of varied size groups from two to four students with their desks pushed together for easier communication and sharing. For the most part, students appeared to work with the same group of students in all of my observations. During group work time, the teacher was responsive to attending to all student groups. The teacher explained to me that as students worked together in groups she finds herself “running around everywhere” to support students to complete each day’s mathematical task. I observed the teacher doing this in each of the classrooms in all of my visits. As such, students relied on the teacher during most of the group class time to successfully engage in and complete mathematical tasks. The teacher explained she expected this dependence on her during group class time. Although students were provided opportunities to work together, the majority of interaction was between teacher and student.

In my observations, typically class began with the teacher either using direct instruction techniques to teach students new mathematical concepts or skills, and/or explaining the mathematical task she intended for the students to engage in during the class period. The direction the teacher took at the beginning of a class period appeared to
be dependent on whether the mathematics lesson was new to the students, was intended to pull together ideas from past concepts that students had already been exposed to (taught), or a class period intended for students to practice what they already knew.

Many times, the teacher would conclude the teacher-directed aspects of the class period with asking assessing-type questions, before asking students to work together. Assessing-type questions used by the teacher were ones in which the questions were designed to help the teacher understand students’ thinking on concepts relevant to the day’s mathematics lesson. For example, one day’s lesson involved a task in which students were asked to show multiple representations of a linear relationship (the graph, equation, and table). Before releasing students to work on the task, the teacher asked the students this assessing-type question: “What does the slope stand for?” After a few of these assessing-type questions, and some assurance that students understood the material, the teacher released the students to, as the teacher said, “work together with your group on the task.”

This lesson structure of introducing the task of the day, assessing-type questions, and then followed by opportunities for students to work together, was the norm and practice in all but one of my 15 classroom observations. However, providing opportunities for students to work together did not always yield students effectively working with their peers to complete the task. Many groups of students did not immediately engage in the task, talking about things unrelated to math, or just hanging out. Other students independently began to work on the task without conversing with their peers in the group. During nearly the entire group work portion of the class period, about two-thirds of the class time, the teacher provided student support, going from one
table to another helping the students begin the task, providing explanations of what to do next, and/or explaining where they were correct or incorrect, and providing redirection as needed.

For example, one day students worked on a task called “Trip to the Zoo.” The task presented this situation: The zoo charges $4 admission fee along with a $2 charge for each exhibit you visit. Students were asked to create an equation, a graph, and a table to model the mathematical relationship. The students were also to state the initial value and rate of change. As the teacher traversed from table to table supporting her students, it became clear that simply opening up opportunities for students to work together, did not necessarily provide students with opportunities to successfully understand and complete the task. The following is one sample of a teacher and student exchange as the teacher worked to provide support during the zoo task.

S: Is this right?

T: No, erase. Let’s talk about what’s going on and how to think about it. If we went to the zoo and visited one exhibit, how much will you pay?

S: What do you mean I got it wrong? I did the exact same thing as you [as the teacher].

T: Erase those.

In this exchange the student was unsure of the correctness of his answer. The student attempted to complete the task based on what the teacher had provided during her explanations of the task and the assessing-type questions; however, the task was not exactly like the one presented to the class during the teacher explanation, and instead asked the students to put ideas together which used ideas provided by the teacher, but was
not exactly like the problem the teacher presented. The student struggled to put ideas together, to think beyond the example, and instead required additional scaffolds from the teacher. Additionally, it appeared the student expectation of how he was to engage in the task was to simply copy what the teacher had done.

In many of classes I observed, and for much of the group class time, students had their hand raised or were waiting for the teacher to stop by and provide direction and assistance. When a group of students came to a point where they were not able to move any further in completing a task, they would stop working and wait from the teacher to intervene for support. Although students conversed with one another during group work time, for the most part, the students were dependent learners, relying on the teacher’s support to complete a task, sitting passively and waiting when they were stuck and needed the teacher’s assistance. The notion of dependence on the teacher observed in this study did not simply exist in the student need for teacher support, the teacher appeared to hold expectations of her students that they needed her to complete tasks and develop mathematical sensemaking.

**Teacher expectations and student dependence.** The teacher’s expectation of the students’ ability or willingness to participate in mathematics can be understood in relationship to the students’ dependency on the teacher and vice versa. The teacher recognized the students need this kind of support. She explained that students are, for the most part, not capable of engaging in the mathematical tasks on their own. The teacher notes, students need “someone actually going step by step . . . having someone look over them . . . they need someone to constantly be by their side.” The teacher beliefs about
students needing her “by their side” was exactly what was observed in my classroom observations.

Of the 15 observed lessons, most of the support students received during their group work time were similar from group to group. On occasion the teacher would ask students to stop working in groups so she could clarify an instruction or process based on what she observed students struggling with. However, the teacher stated that “half of the time they [students] don’t even know what is being asked.” The teacher noted she is sometimes perplexed because even when the steps are, in her mind, “right in front of them . . . they’ll just stop [working].” The teacher further explained the challenge “when the material has multiple steps, they’ll just stop.” The teacher appeared to struggle with knowing how many steps to provide to students and how many steps are too much such that students become overwhelmed and do not engage in the task.

In an interview, the teacher recounted a specific task involving the mathematical idea of line-of-best-fit in which many students did not engage. “Line of best fit” is a line that best represents, makes sense of, a set of data that appears in a scattered format of data points on a coordinate plane. The teacher believed the task was scaffolded enough, with steps clearly defined, such that students would be able to successfully work together and problem solve. However, the students were still not able to engage.

[At first] they were doing really well with histograms and dot plots [pre-work students engaged in to begin to understand ideas of graphically representing data]—they were like, “I have my data right here”—they were able to make their graphs . . . but, [creating the] line-of-best-fit, they felt like there was a whole bunch of different components. There was a graphic organizer and it was labeled
step one, step two, steps three and they stopped [working]—they felt like it was overwhelming—those that didn’t ask questions, just gave up . . . [the low students] just disconnected.

The teacher was hopeful that she had given the needed scaffolds for student success, yet students were still not able to successfully complete the task. The teacher acknowledged that language access might be part of the problem, but also indicated that native English-speaking students did not readily engage in the mathematics tasks either. On occasion, when students did engage in a task and ask thoughtful questions, the teacher expressed that it can be a delightful surprise, “I have a handful of students who will actually ask questions where I'm like, Oh, like I was totally not expecting you to ask me this.” This surprise may speak to the expectation the teacher has for students to be able to problem solve without her direct involvement.

The teacher shared an example of a student engaging in a mathematical task, asking thoughtful questions that surprised her.

We were talking about discrete and continuous functions, and there were two different word problems. One story was about a student mowing the lawn and how he only got paid only if he mowed the whole lawn. The other story was a student who worked at the grocery store and got paid hourly. I had one student ask “how does that work—if he doesn’t have homework over the weekend and mows 10 lawns but doesn’t finish the 10th lawn—is he still going to get paid the same amount?”

The teacher went on to say,
I was mind blown . . . for that student to make a connection—the question wasn’t even part of the task. I was like—hold on . . . that’s a really good question. So, I then brought it up to the whole class and we had a really nice discussion and made connections, I saw positive attitudes—which makes me feel good. The interesting thing was that this student is shy and has a 504\textsuperscript{10} and struggles in math.

The teacher appeared surprised, yet delighted when that student was able to engage in a task and ask thoughtful questions. For the most part, teacher expectations and student engagement went hand-in-hand. There are implications for teaching practices in terms of the amount and type of scaffolds students need to engage successfully in tasks. Opportunities for group work were evident but understanding the interplay that surfaced between teacher expectations and student dependence on the teacher is complex. The teacher expected students would not independently engage in tasks, and that is exactly what students did. For the most part, students appeared to be dependent on the teacher during most of the group class time, waiting for the teacher to further scaffold and support a step-by-step process. The teacher expected this behavior even when she believed the appropriate amount of scaffolds were provided for students to successfully engage in and complete a task. The interplay of the teacher’s expectations of the students’ willingness and ability to complete mathematical tasks and students’ dependence on the teacher for mathematical support raises questions: Do students align to the expectation level of the teacher? Or does the teacher’s expectations align to student needs? Further,

\textsuperscript{10} A 504 is a plan in which a student who has an identified disability under the law and receives accommodations that will ensure their academic success and access to learning.
how much of a role do scaffolds and the way in which this group work was structured play in this teacher’s expectation/student dependence interplay?

**Factors that Interact as they Influence Student Willingness to Participate**

Through conversations from the teacher interview and the student focus group, both the teacher and students expressed productive beliefs about the importance of classroom participation and the need to ask questions to enhance learning. There were several factors observed in the classroom that seemed to either support these productive beliefs or challenge them, and as such, influenced student willingness to participate. These factors included a classroom atmosphere conducive to students taking mathematical risks, student participation in small group versus whole class activities and discussions, and student labeling and peer support during group work discussion. In the following paragraphs I describe these influencing factors, the ways these factors played out in the classroom, and how these factors seemed to influence and be influenced by one another and ultimately impact student willingness to participate.

**Classroom atmosphere.** Student beliefs and understanding that they need to participate in class to understand mathematics and their willingness to participate seem to be in conflict with one another. Classroom atmosphere appears to influence student willing to participate. More specifically, students in the focus group explained they realize the importance of asking questions to develop mathematical understanding yet are reluctant to do so. Several students stated that when no one in the class is answering the teacher’s questions, it creates an atmosphere that discourages them from participating. One student stated, “if its awkward, I don’t think anyone would want to raise their hand.”
Another student went on to say, “But if everyone is talking about it [math ideas], giving different opinions, then I think more people will say this and they [will participate].”

This awkward atmosphere around class participation was witnessed on occasion in my classroom observations. During the opening of a lesson when the teacher asked students questions to provide a grounding of the mathematics that would be used that day, the teacher would frequently resort to calling on specific students since students were not willingly participating when a question was posed. Students may have interpreted the environment as one in which answering was mathematical risk-taking. At times, one or two students would raise their hands as willing participates in classroom discussion. At other times, students articulated reservations to participate. In one exchange when students where not participating, the teacher called on students by pulling out popsicle sticks with student names on them. One student said aloud “No! What if my name is called!” During another lesson, the teacher asked students to take turns showing work on the board at the front of the classroom. Once a student had shown some work the student was to pass the whiteboard marker to another student for them to take a turn at the board showing their work. The body language of students indicated an unwillingness to participant as evidenced by students’ heads or eyes lowered and their hands at their side or crossed over their chest. One student said “Don’t you dare give me that marker.” Another student shook her head “no” when handed the marker, but then reluctantly came to the board and was successful. In the student interview, however, many students articulated they recognize the importance of participating and asking questions to learn. This student nicely captured this sentiment:
When students laugh at a student answer, it might discourage you from asking questions. In math, you need to ask questions to make sure you fully understand the concept. Because if you don't raise your hand and you just keep going through with it, eventually it's going to come up again and again and you're not going to know it still.

This student’s sentiment about feeling reluctant to ask questions in class for fear of being laughed at by other students, even though the student recognizes the importance of asking questions to gain clarity, brings to light concerns about classroom atmosphere and whether being wrong in mathematics class is okay or not. There appears to be a relationship between students’ desire to participate and their willingness to engage in front of peers for fear of being wrong.

Fear of being wrong. Students fear of being wrong in class appears to influence their participation and may also impact the construction of their math identity. In the student focus group interview, one student stated, “If you say something not right, you will be embarrassed to speak out, because when you say something wrong, they [peers] embarrass you.” Students have developed attitudes about willingness to participate from prior exchanges in the classroom. During classroom observations, I witnessed students making comments that did not promote the production of a positive mathematics identity. For example, the teacher asked a student during a whole class discussion, “Which is bigger, 2 or negative 2.” The student answered “negative 2.” Many students in the class laughed aloud. The teacher typically did not address these types of classroom behaviors aloud to the class. She may have addressed these behaviors on an individual basis at other times.
As with the students, the teacher also acknowledged the classroom atmosphere and particularly peers can influence students’ willingness to participate. The teacher noted that “students don’t want to look dumb in front of so and so.” The teacher indicated she believes students’ un-willingness to participate in class has to do with fear of being laughed at and not because students do not believe they can be successful in their answers to questions in mathematics class. The teacher articulates this idea through the following quote.

It’s not that they [students] think they cannot do [mathematics]. I mean if they got the wrong answer, that’s no biggie. But I think it goes, I don’t want to get laughed at if I get the wrong answer, which I told them to take care of yourselves because I mean kids are mean.

The teacher always supported and redirected incorrect answers for the students to put mathematical ideas together as classroom discussions occurred. The teacher indicated students may have the mathematical skills, but that peers influence their unwillingness to put themselves out there for public scrutiny.

Not all comments made by students during my observations inhibited the construction of a positive math identity. There were several incidents whereby students clapped or made a “good job” comment when another student successfully answered a question. However, student comments that supported the construction of a positive student mathematical identity were more often heard during small group conversations instead of whole class discussions. Students acted as though working in small groups was a safer environment for risk-taking than whole class discussions, and as such, were more willing to participate.
**Group size.** Students appeared more willing to participate during their group work time rather than during whole class discussions. Student comments in the classroom were either ones that appeared to promote the construction of a positive math identity or inhibit it. It is interesting to note that wrong student answers provided during whole class discussions, were many times greeted with laughter or other nonproductive language from students likely influencing the development of their mathematic identity. However, student comments made between students during small group work were more encouraging and affirming of student efforts and therefore, offered the opportunity for students to construct a positive math identity. In the following student exchange between four students sitting in a cluster of desks pushed together, I witnessed students affirming correct answers and working to make sense of the mathematics together.

S1: You can’t change the story, it is already done. We have to work with the story.

S2: But when the story says (student correctly explains the relationships shown in the table as it relates to the story).

S1 and S3: Yeah—that makes sense.

S1: Did you plot the points?

S2: Yeah—it’s easy (student proceeds to show students how to plot the points).

S4: I thought the reason the point is placed here (points to graph) is because we were looking for where it crosses the axis.

S1: Oh yeah—I see. Come on S2 even I’m not smart and can do this, you can do it . . . I don’t know how to do the next one.

S4: I also don’t know (however, students continue to work together).
S4: It can’t go to 100, right?

S1: Right.

Since the teacher provides opportunities for students to work in groups each day, exchanges such as this were made possible and at times were witnessed. Although, most commonly, as noted previously, groups struggled to work productively together unless the teacher visited the groups to answer questions, redirect discussions, and provide support for student success.

The productiveness of peer support to understand mathematics was spoken about during the student focus group interview. Students indicated their peers did not help them much toward understanding mathematics. Students explained that although they may work together, they do not believe students actually help other students understand math. This view may be related to the lack of group work structures or protocols in the classroom. As one student indicated, “Some peers feel like it is better for them to just give you the answers, but I feel like that isn’t very helpful, instead of being like, hey this is how you do it.” The following dialogue between a small group of students with the teacher shows how students provide answers but not actually help their peers to understand the mathematics.

Teacher to S5: You have the right answer.

S6 to Teacher: What do I do next?

Teacher: S5 help S6 (The teacher moved to support another group of students).

S6 to S5: What do I do?

S5: Add 5.

S6: Here? (S5 moves his paper over to S6 so S6 can copy down S5’s work)
This type of exchange between students, whereby students were helping each other find answers and not developing an understanding of mathematics, was a common practice. During classroom group work, the teacher constantly assisted students when they stalled in their work or needed direction on what to do. The teacher indicated there is a tension for her between spending too much time with any one student or group and students self-labeling themselves as not good at math. The teacher fears this self-labeling might influence the student’s confidence.

I feel like staying too long or lingering too long on a specific student, then you know right off the bat, students might think they’re dumb, so I try to go from group to group and open it up to the group. I tried working with individual students, but got attitudes back like “oh, she’s coming back here because I didn’t do well,” so I tried to cut back from that just so I won’t shoot down their confidence even more.

The idea of students fearing too much attention from a teacher during class was addressed by the students in the student focus group. A student implied that if a teacher has a “gut feeling that the student is struggling, they should go up to them and help them.” For a teacher to get this “gut feeling,” the students indicated the importance of a teacher really knowing them and appropriately intervening when necessary. This importance of relationship-building between the students and the teacher is another theme that emerged in this study and is addressed later in the paper. When and how a teacher intervenes to support student learning, such that confidence is developed and not diminished, is a component of good teaching practices. This practice requires knowing
how best to support individual student needs on a socioemotional level and an academic level.

**Labeling and peer influences.** The teacher articulated that the fear of being “labeled” and peers exerts a strong influence on students’ willingness to participate, their effort, and ability-beliefs. The teacher indicated students label themselves and others label them too. The teacher recanted a classroom discussion whereby students believed they already knew which students would score the highest on an assessment. Before the assessment began the teacher told the students that the top three students would receive extra credit. The students “started calling out who they thought would get extra credit – I told them, you don’t know that who it will be.” The teacher said that the students were actually wrong in their predictions. She reiterated to the students that “if you put the effort in, you will come out on top [as one of their classmates did].”

The teacher indicated that students will go along with a group and not participate even when she knows the student could participate with a thoughtful or useful idea. The teacher noted, “I have one student who likes to participate. Just that when it comes time to it, he kind of chokes just because there’s some other students that he wants to impress.” The teacher expressed how she tries to help students overcome this idea of labeling and peer pressure with messages to her students like “I tell my students don’t pay too much attention on being the cool kid, whether it might be not getting good grades on purpose because that’s what our group of friends does and stuff like that.”

While acknowledging the merit of what the teacher expressed, a student in the focus group indicated if a student possesses confidence in themselves, they can overcome much of the peer pressure and participate in class in a way they know is
needed for success. Ignoring peer pressure completely as a ninth grader may be extremely difficult. The student noted in the following quote the role confidence plays in deciding to participate.

It [participating] depends on how you feel about yourself. Like you're either antisocial or social, whether you're able to open up to people and raise your hand and let the teacher know that you're not understanding. Or if you'd rather just be there and be like, I don't want to be the person that feels like I don't understand the work and be embarrassed about it. So, it depends on how the person feels about themselves, opening up.

This student’s comment indicates that he believes there are inherent social behaviors that shape a student’s willingness to participate or not in class.

The teacher affirmed the idea that there are social reasons that influence students’ participation and their willingness to work through the difficult task of exposing their knowledge or lack of knowledge to their peers to learn. The teacher contextualizes these social reasons as social skills. She explained that she tells her students:

Some of you guys don't have the social skills to ask questions or ask for what you want or what you need—so, [let’s] start this in the classroom. So, I'm trying to get them comfortable with asking for what they want because we all need those social skills.

The teacher and students seem to indicate that desired social skills both inside and outside the classroom are ideas of agency; the ability to ask questions and take actions on your behalf to achieve a desired outcome. The teacher believes that students’ beliefs about themselves as capable learners follows them into high school. The teacher articulated,
I would say that it [belief in oneself] follows them from elementary and having that negative mindset of not being a good student or incapable of learning math or I’m incapable of working out certain problems—that [belief] follows them.

In general, the classroom atmosphere was one in which on the one hand students and the teacher seem to understand the importance of student participation in the classroom for student success, but on the other hand, recognized that the current atmosphere hindered student agency and their willingness to participate for fear of being wrong. Students lacked the strategies to negotiate the competing dynamics.

Students used language that suggested how the construction of a positive student identity occurred in small groups, yet they felt their peers did not support their understanding of mathematics. Students recognized their confidence plays a role in their willingness to participate, yet peers and the fear of being labeled impeded access to additional knowledge. As such, there was a complex relationship between productive beliefs held by the teacher and students, and practices taken up in the classroom setting.

**Relationships and Relatability Matter**

The teacher and students indicated that the teacher-student relationship matters for student success and may impact confidence. Further, the teacher was raised in the same small town as the students, attended this same high school, and is Hispanic as are most of her students. The teacher explained how she uses relationships with her students and her relatability of culture and community to support a productive student identity and to encourage her students to work hard to achieve their goals at school and in life. Additionally, the teacher also recognizes the importance of working to support student learning by providing opportunities for students to align the mathematical content in
ways that may be more relatable to them. In the following paragraphs I outline this relationship and relatability theme that emerged from teacher and student interviews and classroom observations, showing why relationships and relatability are offered as opportunities to influence student confidence potentially impacting student success.

**Getting to know students.** The importance of teacher relatability to students surfaced by students in that students find it important for the teacher to take the time to get to know them to meet their academic needs. The following student quote demonstrates a belief that a stronger student-teacher relationship will improve grades. This seemed to be the sentiment from most students in the focus group:

> I think the teachers should take more time to learn about the students. If the teacher will take the time, it's a lot more work . . . but take the time . . . grades could improve from doing that. Instead of the teacher saying, here's the work, this is how you can do it.

This student comment seems to point to a connection between not only teachers getting to know students so their grades can improve, but also that teaching practices might be impacted by this relationship in such a way that classroom interactions would be more about learning and not as much about getting work done, that is getting through material.

Students further indicated the importance of teachers getting to know their students so that more productive peer work could be actualized. As noted previously, students felt their peers, for the most part, were not very helpful in supporting their learning of mathematics and instead were more useful in just providing them answers. They suggested if teachers got to know their students’ style of learning, teachers could create purposeful student groups based on similar student learning styles and peer work
might be more productive. One student indicated, and others agreed that “assigned seat groups where you sit with your group that does the same format that you do [student shares similar learning styles] so you guys can work together and then, another group that has a different format or a different way of learning than they do [would be grouped together].” According to these students, understanding student learning styles could create more productive student collaboration.

Another factor that surfaced to explain the importance of teacher-student relationships, was that through relationship-building there might be a heightened awareness of factors from outside of school that are influencing student learning. Students explained that if they were comfortable talking with teachers, teachers might gain a better understanding of why students are performing the way they are. The teacher recognized these influences and noted it is important to communicate and understand these influential factors in the students’ home-life to support students academically.

Sometimes your [students] are just having a bad day and if we don't ask why they're not doing the work or what's going on, what might be going on at home, we'll just give them the F without fully understanding their background or what is actually going on.

During the focus group discussion, students corroborated what the teacher was saying about outside influences on student success. One student indicated students might actually know the mathematical material, but because of what is going on at home, they are not able to show what they actually know in class. One student explained, “maybe their grades are bad just because they don't know the material [or] maybe they do [know the material] and they're really smart, but things at home might not be the best.”
The teacher indicated that she felt that when the relationship with a student is good, it transfers to student motivation and success in the classroom. The teacher said that when there is a productive student relationship, students “don’t want to disappoint” her and appear to care more about being successful, trying harder and engaging in tasks.

Classroom observations indicated that this teacher put forth a daily concerted effort to develop relationships with her students. Each day, the teacher greeted her students at the door, personally welcomed each student to class. The classroom atmosphere was one of respect between the students and teacher in that when the teacher addressed the class, all students would pay attention to what was being said. I rarely heard any student or teacher speaking over the other and instead, students and the teacher waited to respond until the other finished.

Near the end of the first semester, the teacher indicated that she felt that her students’ confidence had increased. I asked her what she believed contributed to this student increase in confidence. The teacher indicated that she believes student confidence has increased throughout the semester in part due to the relationship she was building with her students.

[When first semester started] they had no confidence whatsoever except those few students who were achieving their goals but the rest had zero, or at least in my class, zero confidence. [Now] they are building confidence, I would like to think it is me actually trying to build a relationship with them because if I'm just up there talking and you know, disregarding whatever they're asking me, then there's no relationship there. They're going to be intimidated by me. . . . They're going to
be scared to ask me questions. So being available, being there for them whenever I can, helps build that confidence.

The students and teacher acknowledged the importance of a productive relationship. It is apparent that the teacher is working toward building that relationship with her students through her daily interactions. However, since the second semester academic outcomes were not what the teacher had hoped for (see Table 1) and remains a problem suggesting that academic outcomes are clearly a product of many factors, and it does not simply sit in relationship building.

**Early warning communication.** The teacher’s views on the importance of the student-teacher relationship coincides with students’ views on student-teacher relationships particularly in terms of the important role that communication plays in their academic success. In the student focus group, many students stressed the need to have a relationship with their teacher whereby they would feel comfortable asking questions and letting the teacher know when they are struggling and needed help. The students stressed that this communication goes both ways – the teacher also needs to communicate with them and their families as soon as they see a problem rising and the students need to inform her when they are having problems. In other words, through this student-teacher relationship, early warning communications can facilitate productive teaching and learning. This student’s opinion captures this idea, “Just keeping the communication going, keep checking in on them [teachers checking in on students]. Maybe after one week you'll see where the grades [are], if they lost points or if they gained. Just keep in contact with students.”
The students stressed the need to have a student-teacher relationship such that they are comfortable to open up to their teachers. One student stated, “When we’re really struggling, if we feel like we are not good enough and we need to [be able to] tell the teacher.” Having the relationship and comfort to “be able to” communicate, as one student said, “speak my concerns” with teachers, provides the teacher with the opportunity to intervene and support student learning in a timely manner.

Another aspect to open communication and creating a supportive student-teacher relationship is the importance of the teacher’s communication with parents. The students and teacher stressed the importance of having parents involved in discussions about student progress. One student summarized this point by stating “When the teacher's grading and she sees that a student's grade is dropping, they should contact the parent and maybe set up a meeting and talk to the parent and the student together about what they can do to help the student out.”

In this community the majority of parents speak Spanish as their first and many times it is their only spoken language. This means that the school, and teachers in particular, need to find ways to communicate with families so this early communication can occur. The teacher in this study repeatedly emphasized that since she is from this community, went to school at this school, is Hispanic and speaks Spanish, it gives her the opportunity for more fruitful relationship-building and communication with her students and families. In fact, one of the driving forces in the teacher’s decision to come back and teach at this school was so she would be able to form relationships with her students and families to support this community. The teacher stated,
I really liked the subject [mathematics] and I knew I wanted to come back and help my community just because I know that growing up I had those teachers who would just nag on us and didn't actually attempt to make a connection with us or have a relationship with their students. So, I kind of wanted to change that just like my high school teacher did with us. I want to do that with my students and give back just a little bit.

The teacher made it a priority to build relationships with her students believing that through productive relationships student confidence and academic success could be achieved. Students substantiated this belief pointing to the importance of early communication with them and their parents to stay on top of their learning progress. Although a productive relationship between students and a teacher were important outcomes in this study and believed by both the teacher and students to build confidence and support academic success, not all students reached academic success. There appears to be more to academic success than simply having a productive relationship.

**Relatability matters.** There were two ways in which the importance of relatability revealed itself in this study; teacher relatability to students and the community, and student relatability to mathematical content.

**Teacher to student and community.** Not only did the teacher speak to her belief about the importance of productive relationships with her students for building confidence and academic success, the teacher also believes in the importance of cultural and community relatability to her students. She believes that through relatability she provides students with motivation to set and achieve academic goals.
At the start of the school year, the teacher explained that she intentionally communicated with her students that she understood many of the realities that they were encountering because she, too, was born and raised in this exact same small, rural, farming community and went to this same high school. The teacher recalled,

I told my students that my first job when I was 14 or 15 was picking blueberries. I'll hear some of my students say they work in dairies already. They talk about the summer and how they worked in the field to make money for their school supplies or just their school clothes. So, I [tell them] I've been in their position so I understand some of their struggles. That's why I provide all the help that I can give them.

The teacher used her ability to relate to the community in which these students live to be a role model for them, motivate them to set goals for themselves, and to be diligent in their studies. The teacher showed her students two of her work identification badges side by side; one identification badge was from when she was 15 years old, less than 10 years ago, when she was picking blueberries in the local fields and the other was her current work identification badge to teach mathematics at the school.

The teacher also made a point to let the students know that her parents are much the same as many of their parents. The teacher speaks frankly with the students about the need to set goals and work toward achieving them.

I told them, you know, my parents didn't go to high school; they didn't go to college. I know a whole lot about first generation students. So, you have to achieve certain goals and place yourself there - you're not there automatically. No one is given special treatment. I feel like those students who don't have that extra
help [need it], which is why I offer after school tutoring Tuesdays and Thursdays so they could come…and get the help.

The teacher also explained her understanding of families who are in a rural farming community and the impact it can have on student attendance. The teacher spoke about a student who the teacher believes would miss or be late to her first period class because the students’ parents were already at work and couldn’t help to get them to school.

I have this one student, first period, who I want to say missed class like 70% of the time or was tardy…her parents are probably off at work and they don’t have the time to wake her up and get her to school.

The teacher has a high level of empathy for her students which may be due, in part, to her ability to relate as an insider of the community. The importance of community relatability was also evident in the teacher’s recognition of the language needs of her students. The teacher spoke specifically about the English learners she has in her classes and their struggles in navigating both academic and nonacademic language.

Those students who are English learners if they don't have a sentence starter [sentences with blank spaces for students to fill in the appropriate academic terms or vocabulary] or just certain phrases that we see every day, sometimes they don't even know the definition or, if it's slang, they don't really know where to start.

**Mathematical content.** The other aspect of relatability that was important to the teacher and observed in the classroom, was providing culturally relatable mathematical content to students to motivate them to engage in learning. The teacher did this by
including mathematical examples or tasks that she believed to be relatable to the
students’ lives.

Many times, the teacher was observed slightly changing the scenario of a task to
bring students into the task. For example, in the “Trip to the Zoo” task that was explained
earlier in this chapter, the teacher started the task with this statement “Let’s say we go to
the zoo as a class.” The teacher and students began to create scenarios based on which
animals’ students wanted to visit and which students wanted to visit them. Some of the
created scenarios looked something like this:

- Scenario 1: Steven wants to visit the zebras, koalas, and penguins
- Scenario 2: Juan wants to visit the pandas and gorillas
- Scenario 3: Amanda was to visit six animal exhibits

The teacher and students began to discuss ideas about how much it would cost for
each different scenario. Many of the students were highly engaged in this activity and
worked in small groups to think about these scenarios and model them into mathematical
relationships. The students continued this investigation by either creating their own
scenarios of what to visit at the zoo or the teacher would provide scenarios to challenge
them further.

On another observation day, the teacher presented a problem to the students
regarding hosting a football super bowl party. The scenario was about modeling the linear
relationship when groups of 5 people enter a super bowl party together in relation to the
total number of people at the party at any given time. The teacher asked the students to
place this party at their house – in other words, the students were hosts of the party. As
students worked on this task, and the number of attendees started to get into the 100+
party-goers, there was talk about how they would fit this many people in their homes and how big the home needed to be to accommodate this many people. The students appeared to have fun with this activity and were highly engaged. Students appeared to be more engaged in mathematics when these types of mathematical content relatability were enacted. As noted earlier, the teacher still used the practice of visiting student groups to get students to put mathematical ideas together and complete the task, but in the class periods where the teaching practice of placing students into the content was used, students appeared more engaged and interested in the task.

The teachers’ attempts to make mathematics relatable to students and discuss with students her relatability to the community, provided students with motivation to engage in learning.

**Teaching Practices: Variances Between Class Periods**

A final theme that emerged from this study was the variances in teaching practices between the class periods. As a reminder, Class Periods 1 and 2 were students identified as *nearly mathematically ready* for IM1 and Class Period 5 were students identified as *not mathematically ready* with the need for more intense academic support. Given the teacher taught all of these class periods, the same opportunities in all three classes were provided to students for classroom discussions and group work. Some of the observed teaching practices seemed to support student learning and the construction of a positive student mathematical identity and some may have undermined it. The teacher spoke about these variances in teaching practices during interviews. I compare the class periods in the following paragraphs, highlighting student and teacher actions of practices that open discussion for students’ exploration of mathematics.
In all three classes, the teacher asked questions intended to assess existing mathematical knowledge and to promote advancement of learning. The use of both types of questions, especially when used together, can provide students with opportunities to put ideas together and move learning forward (Leinwand, 2014). An example of one of the teacher’s assessing-type questions was: “When the slope is -1/3, how many am I going to be rising?” An example of one of the teacher’s advancing-type questions was: “If I compare two sets of points on a line and then I compare two other sets on the same line, will I get the same slope?” Although these two question-types were observed in all three class periods, the teacher’s practice for implementing these strategies varied depending on the class.

When advancing-type questions were presented in Class Period 5 (the class comprised of students who were labeled as not mathematically ready for IM1) the teacher tended to either answer the questions for the class or switch the questions into a funneling-type question\(^\text{11}\). Table 6 shows a side-by side example of an advancing-type question presented to both Class Periods 2 (nearly mathematically ready) and 5 (not mathematically ready) and the ensuing teacher and student dialog.

Table 6.

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{Class Period 2} & \textbf{Class Period 5} \\
\hline
T: How could (the idea of) opposite numbers be related to absolute value?  
(Silence in the classroom for about 10 seconds)  
T: Look at this number line & T: How could (the idea of) opposite numbers be related to absolute value?  
(Silence in the classroom for about 5 seconds)  
(There is a number line drawn on the whiteboard at the front of the classroom with 0, 2 and -2 noted) \\
\hline
\end{tabular}
\end{center}

\(^{11}\) In funneling-type questions, the teacher asks a series of questions intended to guide the student to the correct answer instead of pushing students to think more deeply about mathematics.
(There is a number line drawn on the whiteboard at the front of the classroom with 0, 2 and -2 noted) (The teacher gives the students more time) (The teacher calls on a student with his hand raised)
S1: They are the same distance on the number line.
T: What do you mean?
S1: You can see it on the number line.
T: Show me what you mean on this number line.
S1: (Student comes to the whiteboard at the front of the classroom. The student draws arches between the numbers with a marker, as he explains) When I have 2 and -2, I go the same distance from zero.
T: What does that idea have to do with (the idea of) absolute value?
S2: Absolute value is like the number of jumps from zero.
T: Read this definition and see if this makes sense with what we are saying (teacher presents a definition of absolute value on the whiteboard)

(The teacher points to the number line) T: Which is bigger 2 or -2?
(Silence – no student hands are raised) T: Nod your head if 2 is bigger than -2
(There is an awkwardness as most students nod their head) (The teacher asks one student) T: Which is bigger 2 or -2?
S1: -2
(Students laugh 12 )
T: I think you meant 2, right?
T: So how far is 2 from 0?
(The teacher points to the number line on the whiteboard and shows the movement of two spaces from 2 to 0)
S2: 2
T: How far is -2 from 0?
S3: 2
T: Great - read this definition to see how absolute value and numbers that are opposite are similar (teacher presents a definition of absolute value on the whiteboard)

There were several differences between these two class exchanges that may account for variation in student mathematical understanding and the construction of student identity. First, in Class Period 2 the students were provided a longer opportunity to consider the posed question. In other words, the time between a question and the teacher intervening was longer in Class Period 2 than in Class Period 5. Second, in Class Period 2 the teacher asked a follow up, advancing-type question, “What do you mean?” giving the student a chance to make sense of his understanding, whereas in Class Period 5 the students were asked an assessing-type question, “Which is bigger 2 or -2?” Not only

12 It is not clear if the student who answered the question was joking by giving this answer, didn’t hear the question clearly, or did not know the correct answer. The student appeared uncomfortable by the interaction.
does this assessing-type question not necessarily advance student thinking to the relationship of absolute value and numbers on a number line, the idea of which is bigger 2 or -2, is a concept that is developed in early middle school and is not aligned to a freshman mathematics course. This below grade-level type assessing question was likely used by the teacher because she was trying to understand if students at least understood magnitudes of integers, but she did not follow with a question bringing students to the conceptual understanding of the grade-aligned mathematical concepts, absolute value.

Third, a student in Class Period 2 was asked to come to the whiteboard to demonstrate his understanding, whereas in Class Period 5, the teacher demonstrated the distance from the numbers to zero. Active student participation is important to build student understanding and agency (Moschkovich, 1999). Fourth, in Class Period 2 there was a mathematical connection made during whole class discussion which begets the overarching idea that the teacher intended for the class, “Absolute value is like the number of jumps from zero.” However, in Class Period 5, students were not provided the scaffolds to get to the overarching idea and instead the students were to make that connection on their own by examining the definition of absolute value which was provided for the students on the board.

The exposed differences in teaching practices between Class Period 2 and 5, such as wait-time, active student participation, types of questions and questioning techniques, and connection-making to the overarching mathematical ideas were observed on many occasions. The teaching practices observed in the Class Period 1 (the other class population of students identified as *nearly mathematically ready* for IM1) were more aligned to the practices used in Class Period 5, although the practices were not as stark a
distinction as was observed between Class Period 2 and Class Period 5. The teacher recognized these differences in her classes.

Every single class has students who get it and ask questions. First and second periods, they’re willing to work together and talk, fifth period is not. Fifth period, I have to be a little stricter. First and second period I find myself joking around with them. Fifth period it gets really hard to steer away from what needs to happen.

The teacher pointed out that she recognizes that students in her first two class periods appear to work together more productively than in her fifth period class. Further, she states that she needs more strict interactions with students in Class Period 5 than the other class periods for fear of losing control of the students’ attention and not being able to complete the day’s lesson. I asked the teacher why she thinks this is that she struggles to keep students engaged and fears losing control with students in Class Period 5. She indicated that she is unsure, but speculated that “some students have stronger personalities in that class, I guess, and it’s just really hard [to keep them engaged and on task].

Student engagement was another factor that the teacher differentiated between her class periods. “Second period we’ll add ourselves into scenarios and stuff like that; thinking outside of the box and looking [at] ‘what if.’ However, in the other class periods there is a fear of losing control and not being able to bring the students back into the lesson, “I am often tempted …stay or steer away…because they [students] will ride the wave and not come back.”
Students were also observed in Class Periods 1 and 2 sitting in small groups, whereas the students in Class Period 5 sat mostly by themselves at the start of the class period. When it was time for Class Period 5 students to work together, the teacher needed to ask students to get with their partner; this was not necessary with the other two classes since students were already sitting in small groups.

Another observed difference between class periods is the frequency of students asking to be excused from class to use the restroom. Throughout the 15 classroom observations, only one student in Class Period 2 asked to be excused to use the restroom and no students in Class Period 1. However, in Class Period 5 it was common for a student to ask to leave the classroom about 10 minutes into the class period. Then, there was a steady stream of students leaving the classroom one after another, as one student returned the next student would leave. One student in Class Period 5 even used the restroom as a reward for work accomplished, “Hey my boy got it right – can I go to the restroom now?”

Lastly, in nearly every observation, the teacher and students in Class Periods 1 and 2 engaged in the day’s lesson, conversing and problem solving, for the entire class period, usually running out of time due to the class period bell ringing. Whereas in Class Period 5, many times the class appeared to complete the lesson about 5 minutes before the class period ended. Then, the students would have their backpacks collected and ready to leave the classroom as soon as the bell rang.

Through observations and conversations with the teacher it was recognized that teaching practices varied between the three class periods. The teacher articulated her struggle to implement teaching practices with her Class Period 5, practices she readily
was able to do in Class Periods 1 and 2. Instruction in Class Periods 1 and 2 opened up discussion and student exploration of mathematics, which may lead to student success and engagement. These practices were more readily observed in Class Period 2 than in the other two classes.

**Qualitative Summary**

In this section of paper, I examined the experiences of students and the teacher in the three class periods. I focused on opportunities provided to students to construct a positive student identity and academic success and the factors that supported or undermined it. I explained from interviews and observations the practices the teacher used to motivate classroom participation and how classroom activities may be contributing to student success and the construction of their mathematics identity. These practices included organizing students in small groups, providing a variety of assessments and asking various types of questions to check student understanding. I examined how the teacher’s focus on relationship building and attending to student relatability in math content, and encouraging student demonstrations of understanding and supporting math connection-making to solidify mathematical concepts influenced students experience in the classroom.

Although opportunities to engage in learning were evident in all classes, instructional practices, what the teacher did as teaching and learning were occurring, varied between the classes. Practices that potentially impacted the construction of a positive mathematical identity and academic success were more readily seen in Class Period 2 then in the other two class periods.
Conclusion

Although there was an overall statistical significance in student academic growth in this study, not all individual class periods’ growth was statistically significant. Class Period 2 was the only class which showed statistically significance in growth, the class period in which observed teaching practices seemed to support the construction of a positive mathematics identity and academic success. This was the period where the teacher was observed providing adequate student wait time, advancing type questions, opportunities for students to demonstrate their work, and whole class mathematical connection-making. The math attitudes and perceptions survey revealed that over half of student responses showed students held productive mathematical dispositions. Class Period 2 showed the highest percentage of productive mathematical disposition responses.

Various factors seemed to support or hinder the construction of a positive student identity and academic success. These factors included grades, student dependence on the teacher, teacher expectation, classroom atmosphere (fear of being wrong), peer interactions, teacher and student relationships, teacher to student relatability and student relatability to mathematical content. These factors appear to exist in a complex interplay, influencing one another. Additionally, this interplay appears to influence another interplay consisting of student confidence, willingness and motivation to participate, and academic success, which also seem to influence one another. Taken together, these two interplays appear to live in an inextricably interrelated network of influencing factors which influence and are influenced by one another. As this interaction occurs in the
classroom, teaching and learning are impacted and influenced. These influences support or hinder the construction of a positive mathematics identity and academic success.

Further, the teacher and students hold productive beliefs yet at times teaching and learning practices in the classroom did not align to those beliefs which may have hindered the construction of a positive mathematical identity and academic success. Lastly, teaching and learning practices in Class Period 2 were more aligned to the teacher’s and students’ productive beliefs than in the other two class periods.

In the next chapter I discuss these qualitative and quantitative results and the idea of this inextricably interrelated network of influencing factors and the impacts and influences on the construction of student identity and academic success.
CHAPTER 5
DISCUSSION

This case study was conducted in the unique context of low-tracked ninth grade mathematics classes in which the teacher used a reform curriculum. The reform curriculum sought to address unfinished\textsuperscript{13} mathematical learning from previous courses and prepare students for success in their next mathematics course without need for remediation. The overarching research question was: How do students’ experiences in the reform course influence the construction of their mathematics identity and impact their academic outcomes? The following guiding research questions also guided this study:

1. How are students’ beliefs about their mathematics ability constructed in the reform class?

2. What factors are supporting or undermining positive mathematics identity and how is math identity related to academic progress in a reform course?

Due to the complexity in the construction of students’ mathematical identities (Martin, 2000; Aguirre, et. al., 2013), and to develop an understanding between identity and academic outcomes, this study employed a mixed-methods design. Data was captured through teacher and student interviews, numerous classroom observations, and pre- and post-academic assessments. Students’ mathematical attitudes and perceptions were captured in a survey. Data was initially organized, analyzed, and triangulated around the guiding research questions. As themes emerged, data was synthesized into four findings:

\textsuperscript{13} Unfinished learning is a term used in this study for mathematical concepts or skills that students may have been taught but internalized incorrect understandings, or did not yet learn but are needed in order for access to course/grade aligned standards.
• There is a relationship between academic outcomes and students’ mathematical identities; this relationship exists in an inextricably interrelated network of influencing factors that impacts and influences student willingness to participate and perform;

• All students were provided opportunities to participate and perform in the reform curriculum, but students’ classroom experiences differed between class periods. Students in the class period with greater academic success and a higher percentage of productive mathematical dispositions (Class Period 2) experienced teaching practices more aligned to best practices for developing positive academic identities and success in mathematics than in the other two class periods;

• There was an interplay between teacher expectations, student dependence on the teacher, and student willingness to participate in class that appeared to impede student ability and willingness to persist when faced with challenging tasks in which a high level of cognitive demand was expected; and

• The teacher and students held productive beliefs about teaching and learning mathematics, yet classroom practices and structures influenced discourse and peer group work that impacted student mathematical sensemaking.

In this chapter I discuss the interrelated nature of these findings through the lens of Martin’s (2006) definition of mathematical identity, “the dispositions and deeply held beliefs that individuals develop, in their overall self-concept, about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives” (p. 206). I specifically framed my arguments around the ideas of students’ beliefs and experiences in their ability to participate in class and
perform. The chapter is organized into four sections. First, I discuss the ideas related to beliefs and opportunities as they impact identity construction. Second, I discuss influencing factors impacting the construction of identity and academic success. Third, I discuss the relationship between these two ideas and recommend areas of focus for future investigations on mathematical identity construction. Finally, I speak to the implications of addressing the interrelated network of factors on identity construction as they relate to positively influencing student mathematical outcomes.

**Productive Beliefs and Learning Opportunities**

There is an identified relationship between student mathematical identity and academic outcomes (Blackwell et al., 2007; Martin, 2010). This study supports that notion; however, explanations for differences in students’ academic success and dispositions toward mathematics were found to be complex. Students in Class Period 2 (a student population *nearly mathematically ready* for Integrated Mathematics 1 (IM1)) had statistically significant\(^\text{14}\) growth in academic outcomes \((p = 0.02)\) and a higher percentage of students holding productive dispositions, attitudes, and perceptions \((59\%)\) at the conclusion of the semester. Conversely, Class Periods 1 and 5 showed non-statistically significant academic growth, \(p = 0.115\) and \(p = 0.223\) respectively, and these student populations held lower percentages of productive disposition indicators, 53\% and 52\% respectively. These results indicate a potential relationship between outcomes and identities; the higher the academic outcome, the higher the percentage of productive dispositions; the lower the academic outcome, the lower the percentage of productive dispositions.

\(^{14}\) By convention, the significance level of 95\%, \(p<0.05\), was used for this study.
The complexity in the relationship between student identity and academic outcomes revealed itself in the qualitative results. Students in all three classes all had the benefit of the same teacher and in all three classes the teacher provided students with opportunities to participate and perform in mathematics through whole class discussions and group work. However, as teaching and learning ensued, evidence suggests that the teacher and students took up these opportunities differently in Class Period 2 than in the other two class periods.

Teacher and students’ productive beliefs about teaching and learning, articulated during interviews, were in greater evidence in Class Period 2 than in the other two classes. These productive beliefs included the importance of creating an inviting classroom environment, engaging students in classroom discourse, and cultivating a supportive relationship between the teacher and students. Practices that support these beliefs have shown in previous research to yield positive math identities and this relationship was found in this study as well (Absolum, 2011; Aquirre et al., 2013; Ashcraft, 2002; Boaler & Staples, 2008; Boston et al., 2017; Hiebert & Grouws, 2007; Ramirez et al., 2013; Turner, et. al., 2013; Warshauer, 2015).

Teaching practices observed in Class Period 2 that supported these beliefs included (a) time provided to students to consider posed questions presented to the class, (b) advancing-type questions and dialogue, (c) constructing opportunities for students to demonstrate how to think through a problem presented to the class, (d) making mathematical concept connections during whole class discussion, and (e) making links between intended lesson goals, students’ improvement, and progress toward those goals. For the most part, the teacher was able to implement practices that aligned to her beliefs
and the result was that the teacher and students reacted and interacted in more productive ways—ways that supported the construction of a positive mathematical identity.

The complexity of mathematical identity construction began to unfold as I conducted classroom observations in all three classes. Although the teacher and students appeared to hold productive beliefs and ideas about how best to construct positive mathematical identities, student and teacher behaviors often ran counter to these beliefs in Class Periods 1 and 5. Possessing productive teaching and learning beliefs and providing opportunities to engage in mathematical tasks are key components to the construction of a positive mathematical identity (Martin, 2006) but were not sufficient in and of itself to motivate students to participate and perform effectively in mathematics in these classes. Likewise, students holding productive beliefs about learning mathematics and knowing what they need to do to learn was not enough to push them to be willing participants and to perform effectively.

Opportunities for Group Work

Although students in my study were provided opportunities to work in groups to help support their learning, the groups lacked the structures for student sensemaking to occur. Research advocates that providing opportunities for students to work together and share strategies and ideas about mathematics can create a community of learners and build productive identities and mathematical understanding (Boaler & Staple, 2008; Wenger, 1998). However, evidence from my study showed that providing opportunities for group work is not enough to create a community of learners for sensemaking to occur and build productive identities; there must be structures in place beyond students simply sitting next to each other and a teacher merely instructing students to “work together,” as
was observed in this study. The teacher in this study asked students to work in groups, and for the most part, students chatted with one another. The conversations were more about answer-getting and completing the task than about making sense of mathematics and understanding the interconnectivity between concepts.

Interestingly, qualitative and quantitative evidence from my study suggests that mathematical sensemaking is a very important belief held by the students. In the student focus group, students indicated that the answer-getting dialogue, which typically occurred with their peers during group work, did not help them better understand the mathematics task presented to them. In the mathematics attitudes and perceptions survey, the sensemaking category held the highest percent of productive beliefs (70%). Mathematical sensemaking is a belief that came across as important to students, yet students articulated their frustration about the unfruitfulness of group work toward sensemaking.

The link between mathematical sensemaking, confidence, and interest in mathematics is supported by research. If students rush to do mathematical procedures too quickly instead of taking the time for sensemaking, gaining confidence and interest in mathematics can diminish which can create mathematical anxiety (Ashcraft, 2002; Ramirez et al., 2013). For sensemaking to occur and student confidence to increase, students need time to work through mathematics in a productive way (Leinwand, 2014).

During classroom observations I observed that time was provided to the students with the intent for sensemaking to occur; however, when students struggled with mathematical tasks, students sought support from the teacher, appearing to be dependent on her to help them work through tasks, instead of looking to their peers for support and guidance. The teacher acknowledged that she was “running all over the place” trying to
answer all student questions. The notion that teachers feel a need to “rescue” students as they endeavor to make sense of mathematics, although typically well intentioned, can lead to lowering the student cognitive demand and remove important opportunities for them to make sense of mathematics (Reinhart, 2000; Stein, Smith, Henningsen, & Silver, 2009).

Research supports the idea of the importance of group work structures (César, 2007; Esmonde, 2009; Sung, 2018). As Boaler and Staples (2009) pointed out, part of the success at Railside School was the result of the structures that were put in place for students to effectively and productively work together on mathematical tasks. This was not the case in this study; therefore, student group work did not appear to adequately support student learning. Again, opportunities to work in groups are not enough to promote students’ mathematical sensemaking.

**Opportunities for Discourse**

The teacher provided students opportunities for dialogue and asked high cognitive demand questions intended to move students to a conceptual understanding of mathematics. However, evidence from this study suggests that when students did not participate in the dialogue, as during the in-the-moment teaching, the teacher struggled to move dialogue forward and resorted to questions of lower cognitive demand. Previous research has supported the notion that teachers often have difficulty maintaining a high cognitive demand of their students as students engage in high-level tasks (Stein, Grover, & Henningsen, 1996; Stigler and Heibert, 2004).

Evidence from the study suggests that the teacher was challenged to maintain effective classroom discourse to support a high cognitive demand of her students. This
notion is supported by research, noting that arranging effective classroom discussion is a challenging and complex teaching task (Boerst, Sleep, Ball, & Bass, 2011; Franke, Kazemi, & Battey, 2007). Smith and Stein (2011) have proposed discussion practices intended to support classroom discourse. The Smith and Stein (2011) discussion practices include (a) *anticipating* students’ understanding pathways before teaching occurs so the teacher can prepare a response and redirect learning if necessary, (b) *monitoring* student work to prepare for whole class discussion that brings in student voice and validation of work, and (c) engaging in a *sequencing and selecting* process of student work samples. The teaching complexity plays out in the classroom by knowing which student work sample to present first, second, and so on when classroom discussion occurs. Also, there is complexity in creating anticipatory questions that will be asked of the class to bring student ideas together for mathematical *connection-making*. The need for implementing in-the-moment discourse practices so the teacher is confident and capable of facilitating classroom discussions is important. Further, perpetuating high cognitive thinking during discourse is important for student academic success and confidence. Discourse teaching practice are important aspects in the construction of a positive mathematical identity. Successful in-the-moment decision-making, specific to classroom discourse, typically improves as teachers gain more teaching experience. This study was conducted in a classroom in which there was a first-year teacher. With support, productive in-the-moment, decision-making discourse practices could be fostered.

**Interplay: Teacher Expectations, Student Dependence, and Willingness to Participate**
Evidence from this study suggests there is an interplay between teacher expectations, student dependence on the teacher, and student willingness to participate that appeared to impede student ability or motivation to persist when faced with challenging tasks. The teacher articulated that for the most part her students “need someone to constantly be by their side” for sensemaking to occur, and many are not capable of bringing ideas together on their own. The teacher noted surprise when a student asked thoughtful, probing questions about the mathematics in a task. The teacher’s expectations and notions of student ability appeared to be based on her interpretations of students’ willingness to participate and their dependence on the teacher to engage in tasks. Teaching practices, like classroom discourse, many times are dictated by teacher expectations for students’ success (Boaler, 2015). This interplay between teacher expectations, students’ willingness to participate, and their dependence on the teacher is shown in Figure 3.

Figure 3.
*Interplay between teacher expectations, student dependence, and willingness to participate*
This interplay appeared to establish classroom norms and practices that impacted how mathematics was taught and learned in the classroom. The teacher’s practice of “running all over the place” to support students and the apparent lack of group work structures that support mathematical sensemaking created a classroom structure of student dependence on the teacher. On their own, a dependent learner “is not able to do complex, school-oriented learning tasks such as synthesizing and analyzing information” (Hammond, 2015, p. 13).

This interplay is a troubling outcome, especially in mathematics. The California State Standards for Mathematical Practice (California Department of Education, n.d.) and expected student mathematics practices include the ability to make sense of problems by analyzing information, and at the high school level, reasoning abstractly by synthesizing ideas. This interplay is also troubling for a student population which is predominantly low income and suffering from low achievement. Low income and low achieving students are the population in this study since these students have been called out as needing targeted support and reform in mathematics (Loewus, 2016; NCTM, 2018). The observed interplay impeded student access to analyzing information and reasoning abstractly when faced with challenging tasks. Many students appeared to simply wait for the teacher’s guidance instead of analyzing and reasoning about the mathematics presented in tasks, which affected their success.

The motivation to help students develop a positive mathematical identity is intended not just to help students in school, but also “to change the condition of their lives” (Martin, 2006, p. 206). Supporting students to move from dependent learning to
self-directed, independent learning is imperative for the development of an overall mathematical identity. However, to move students from dependence to independence, Hammond (2015) stated educators need to “empower dependent learners and help them become independent learners, the brain needs to be challenged and stretched beyond its comfort zone with cognitive routines and strategies” (p. 49). There must be classroom norms and practices whereby routines and strategies are in place to foster the cognitive development of independent learners. In order for students to move toward independence, they must be able to persist when faced with challenging tasks. The interplay between teacher expectations, student dependence on the teacher, and student willingness to participate appeared to impede student ability or willingness to persist when faced with challenging tasks.

This interplay, the actors that interacted and impacted one another, affected behavioral outcomes in the classroom and aligned to Bandura’s social learning theory of triadic reciprocal causation (Bandura, 1989). Bandura (2006) argued that individuals interact in communities in which “internal personal factors, behavioral patterns, and the environment operate as interacting determinants that influence one another” (p. 6). Similar to the interplay observed in my study, this behavior theory speaks of the mutual influences between personal factors (i.e., self-efficacy, self-determination, confidence, motivation), environmental factors, and behaviors, as shown in Figure 4.
The interplay observed in the classrooms in this study are like Bandura’s theory of triadic reciprocal causation. Teacher expectations are the *environmental factors* that influence student willingness to participate. Still, there are also *personal factors* (e.g., ideas of confidence, agency, identity, and disposition) that influence *behavioral factors* or patterns of behaviors observable in the classroom, as shown in Figure 5. Student dependence on the teacher is affected by all these factors and affect the students’ ability to complete mathematical tasks.

**Figure 4.**

*A model of the three interacting determinants of human behavior (Bandura, 1986, p. 24)*

**Figure 5.**

*A model of the triadic reciprocal causation of classroom behavior.*
Interactive influencing factors of a triadic reciprocal relationship can contribute to positive or negative behavioral outcomes (Bandura, 2006). Evidence in my study indicates that many students were challenged to successfully perform, indicating that the triadic relationship in the classroom did not yield positive results for all students. Although there was growth in academic outcomes during the semester of data collection, students’ mean correct score on the end-of-semester diagnostic assessment was only 43.3, signifying that students were able to answer only about 43% of the mathematical problems. This is important to consider since one goal of the reform course is to prepare students for their next math class without the need for remediation; however, about half of the students in my study showed a need for further academic support. This raises questions as to how greater attention to group work structures and effective discourse practices could redirect a negative triadic relationship. The redirection might yield greater student willingness to participate, higher teacher expectations of student ability, or more independent learners which could ultimately impact academic success.

**Identity Construction: An Inextricably Interrelated Network**

Additional factors surfaced in my study that seemed to influence the construction of student identity. These factors were not only observed in the classroom (as the interplay factors were) but were also revealed during my conversations with students and the teacher. Evidence suggests that these factors existed as interconnected loops, whereby the factors influenced one another, and when analyzed in aggregate form an interrelated network appearing to impact student mathematical identity. These factors included student confidence, grades, fear of being wrong, and teacher-student relationships. In the
following paragraphs I explain how these interconnected loops interacted with one another and form a larger interrelated network, influencing student confidence and willingness to engage in mathematical tasks, and ultimately impacting student mathematical identity. Table 7 shows these additional factors as interconnected loops and provides an example of how they revealed themselves in this study.

Table 7.

*Interconnected Loops of Identity Influencing Factors*

<table>
<thead>
<tr>
<th>Interconnected Loops</th>
<th>Examples in this study</th>
</tr>
</thead>
</table>
| Grades Loop – Academic outcomes               | **Student:** “If you do good *on* it [mathematics], then you know, you’re good *at* it.”  
**Student:** “If you’re already starting off the first week and you already have an F - it [grades] can either boost or lower your confidence.” |
| Classroom Atmosphere Loop – Fear of being wrong | **Student:** “If you say something not right, you will be embarrassed to speak out, because when you say something wrong, they [peers] embarrass you.”  
**Student:** “[When students laugh at a student answer] it might discourage you from asking questions.”                                                                 |
Although there were three identified interconnected loops of influencing factors, grades, classroom atmosphere, and relationships, these loops do not exist by themselves, instead each loop connects to the others through the confidence factor. Further, two of the loops, grades and classroom atmosphere, not only connect through confidence, they also share the factor of student willingness to participate, again indicating that these loops impact and join other loops. For example, the influencing factors loops in Table 7 shows that both the grades and classroom atmosphere loops interact and are influenced through confidence and willingness to participate, thus connecting all four of these factors together. The relationships loop in Table 7 connects to both the grades and classroom atmosphere loop through willingness to participate, grades themselves, and confidence, further connecting these loops.

The appearance of the interrelated nature of these factors was revealed when students talked about their fear of being wrong in class and described how their behavior was influenced by peer and whole class discussion and group work. This interaction impacted student confidence and willingness to participate, which influenced student understanding of mathematics. Students confirmed that when they lacked confidence they believed this impacted their willingness to participate in class for fear of being wrong,
which impacted their grades because they did not ask questions to understand the mathematics. Figure 6 is a graphical representation of the network of inextricably interrelated identity-constructing factors, interacting and influencing one another.

Figure 6.

Interrelated network. An inextricably interrelated network of identity constructing influencing factors

Research has supported the interrelatedness of these influencing factors. Confidence has been found to have a significant connection and correlation to course grades (Parsons, Croft, & Harrison, 2009) and student willingness to participate and persist (Bandura, 1977). Results from the student mathematical attitudes and perceptions survey and academic outcomes in this study support the notion of this interrelated network. The academic outcomes of this study revealed about half of the students were prepared for their next math course and about half of the students held productive dispositions about mathematics; this data indicates a connection between outcomes and the interrelated network. The survey results showed that the confidence category was one of the least held dispositions by students; only 49% of the productive confidence
indicators were held by students. Low confidence and low performance were therefore also connected in this study.

The interrelated nature of academic outcomes and the seven mathematical dispositions in the survey, including confidence, raise questions about how much confidence plays into the interconnectedness of these influencing factors and how much it impacts academic success. When students in the focus groups were asked what seemed to impact whether they liked math or not, overwhelmingly students stated that grades were the strongest impact but their confidence to participate and “do” math influenced their grades and their willingness to persist. Again, this interconnectedness of these influencing factors is evident.

**Identity Constructing Influencing Factors Inside a Triadic Relationship**

Martin (2000) argued that prior studies have not linked “contextual forces in sufficiently meaningful or complex ways” (p.vii). In my analysis I attempted to understand the contextual factors in a classroom that influence the construction of student identity and impact academic outcomes. The analysis was conducted through ideas about the interrelated nature of the network of identity-constructing influencing factors. Although this interrelated network is complex and possibly viewed by teachers as overwhelming, the interrelated nature of the network could be used advantageously toward positive change in the classroom. I propose that by addressing one factor of the interrelated network, confidence, for example, a teacher could potentially redirect the entire network impacting student outcomes and mathematical identity in positive ways.

As noted earlier, student willingness to participate in math class was one of three factors in the triadic relationship, along with teacher expectation and student dependence.
Willingness to participate was also a factor in this interrelated network of influencing factors. Since willingness to participate was a factor in both, this implies that the inextricably interrelated network of influencing factors is not separate from the triadic relationship and, instead, impacts the triadic relationship through the influencing factor of willingness to participate. A graphical representation of the triadic relationship and interrelated network connection through the factor of student willingness to participate is shown in Figure 7.

Figure 7.

*Graphical representation showing the connection between the triadic relationship and interrelated network through the factor of student willingness to participate*

Student willingness to participate was impacted by both the triadic relationship defined by Bandura and the interrelated network of identity-constructing influencing factors. Attempts to impact classroom practices and structures targeted specifically to any one of the influencing factors might have a positive effect on students’ mathematics
identity due to the interrelated nature of the network. For example, would implementing a student group structure intended to decrease dependence on the teacher be enough to disrupt confidence, impact willingness to participate, and possibly impact grades? Would implementing teaching practices that promote productive classroom discourse, for example, intended to impact student fear of being wrong, be enough to disrupt student apprehension and promote willingness to participate and influence teacher expectations? These considerations prompt ideas for future investigations to understand the construction of productive mathematical identities and academic success.

**Future Investigations**

Understandings and questions that arose as I conducted this study prompted two areas of focus for future investigation. First, there is a need to understand practices and structures to redirect a negative triadic relationship that supports the construction of a positive mathematical identity. Second, there is a need to focus on the pedagogy of mathematical sensemaking that supports a shift from student dependence on the teacher to self-directed, independent learners. The following paragraphs outline these research needs.

**Practices and structures designed to redirect negative triadic relationships.** I propose there is a need for future research targeted at one of the identity-constructing influencing factors to understand practices and structure that might redirect a negative triadic relationship. The theoretical framing of this study recognized that the construction of identity is shaped, formed, and transformed as we interact and participate in a community (Wenger, 1998). The inextricably interrelated network of identity-constructing influencing factors that emerged in this study is shaped, formed, and
transformed in the classroom as students and teachers interact and participate. When a triadic relationship is revealed in a classroom and appears to yield negative outcomes (e.g., nonparticipation or low performance), this impacts identity construction due to the connection to the identity-constructing interrelated network, Figure 6. I hypothesize that if a classroom structure or practice is implemented, intending to redirect an identified negative triadic relationship such as the one revealed in this study, this will impact not only classroom behaviors but also the construction of student mathematical identities due to their interrelatedness.

Several studies have been conducted that examine reform teaching and learning practices for students of color and low performing students (Absolum, 2011; Aquirre et al., 2013; Ashcraft, 2002; Boaler & Staples, 2008; Boston et al., 2017; Hiebert & Grouws, 2007; Ramirez et al., 2013; Turner, et. al., 2013; Warshauer, 2015). My study lends further credence to what many of these earlier studies investigated and found. However, my study sits at a unique juncture in a student’s educational career – the beginning of high school. Like the other studies, my study demonstrates that implementing high quality instruction is complex, but possible and necessary. As noted in an earlier chapter, most California students enter ninth grade not prepared for high school mathematics (CDE, 2019). It behooves educators and researchers to investigate ninth grade classroom practices and structures that support student self-concept, so students can participate and successfully perform while engaging in rigorous tasks. It is especially important to conduct such an investigation when a negative triadic relation is observed.

**The pedagogy of sensemaking: Fostering self-directed, independent learners.**

I propose there is a need to understand student dependence on the teacher that impedes
students’ ability or willingness for mathematical sensemaking. In order to redirect students to be self-directed, independent learners, an understanding of student dependence on the teacher for sense-making is needed. Teaching practices intended to support student sensemaking are not new. As noted earlier in this study, the National Council of Teachers of Mathematics (NCTM) put forth eight teaching practices in their publication *Principles to Actions*. These teaching practices “represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics.” (Leinwand, 2014, p. 9). Additionally, these practices are intended to provide access for all groups of learners and to connect teaching practices to the construction of student identity, agency, and competence (NCTM, 2018). One of the teaching practices in *Principles to Actions* is supporting students productive struggle so they have opportunities to put mathematical ideas together toward sensemaking. Another teaching practice is for teachers to implement tasks that promote reasoning and foster student problem solving. In this study I revealed that the teacher attempted to implement both of these practices, providing tasks and time for students to grapple with mathematics for sensemaking. However, the teacher encountered students who instead of grappling and using the time to problem solve, sought nearly immediate support from the teacher, appearing in the classroom as teacher-dependent and unable to make sense of the mathematics in the tasks on their own. Further, the teacher held the expectation that students needed her support for them to make sense of the mathematics.

In fact, it is likely that the students in my study have never been asked to make sense of mathematics before, and therefore were dependent on the teacher for sensemaking. Research tells us that students in the United States are rarely asked by their
teachers to put mathematical ideas together (Banilower, Boyd, Pasley, & Weiss, 2006; Heibert and Stigler, 2004). Yet, in the reform curriculum that is exactly what students were asked to do, put mathematical ideas together for sensemaking. Although NCTM states that opportunities, tasks, and time are essential teaching practices for sensemaking, teachers are often met by students who are not able to participate from lack of experiences in sensemaking. This lack of student sensemaking experience may promote access barriers for student sensemaking even when the teacher provides the opportunities to do so, and inadvertently create student learners who are dependent on the teacher to complete reform-type tasks.

Additionally, teachers themselves may not possess an understanding of mathematics at the conceptual level, which is essential to move students to sensemaking. Teacher’s own lack of conceptual understanding of mathematics can prevent them from being able to teach students conceptual understanding of mathematics (Manouchehri & Goodman, 1998). Further, as noted earlier, the research school principal noted that her mathematics teachers struggled during teacher training with the conceptual/sensemaking piece of mathematics.

Teaching and learning challenges around student sensemaking are something that education reformers, pre-service teacher programs, coaches, and professional developers need to examine and understand. Understanding the pedagogy of mathematical sensemaking is especially important when a negative triadic relationship exists whereby students are dependent on the teacher to engage in and make sense of mathematics.

The need for research and understanding about the ideas of the pedagogy of mathematical sensemaking and student dependence does not exist in the secondary
educational setting alone. As indicated in Chapter 2, Californian expectations of a well-prepared student entering college articulate eight dispositions students need for success. Two of these dispositions involve this need for students to be self-directed, independent thinkers and doers of mathematics. These expectations state that students need confidence and tenacity in approaching new or unfamiliar problems, and students need to accept responsibility for their own learning (Intersegmental Committee of Academic Senates, 2013). As a society, and specifically as those who are passionate about providing children with options in life, decisive actions toward the work of fostering self-directed, independent students who are able to access and make sense of mathematical concepts needs to be done.

**Implications**

Results from this study prompted several implications for addressing needed shifts in practices and structures that exist in high schools. In the following paragraphs I put forth several implications that emerged from this study that I implore district and school administrators, teachers, coaches, professional developers, and researcher to consider and discuss through the lens of academic success and the construction of a positive student mathematical identity.

**Reform Grading Practices in Secondary Mathematics**

One result of this research that came across loud and clear from both the teacher and the students is that grades matter to a student’s willingness to participate, persevere, and perform. The students indicated that if they received an undesirable grade at the start of the semester, it can impact both their confidence and willingness to participate throughout the rest of the semester. The teacher also recognized that grades can deflate
student persistence and willingness to participate and noticed that many times when a student received a grade on an assignment they simply “crumple it [the assessment or assignment] up,” showing the influence an undesirable grade can have on their self-concept. An implication of this research is for schools to evaluate or re-evaluate and reform their grading practices, specifically in mathematics, due to the impact grades have on the construction of student identity and their performance.

There are several grading practices that schools might consider which could support the construction of a positive mathematical identity and provide opportunities for academic success. A grading practice schools might consider is to provide students with opportunities to retake a test until a successful grade is obtained. Since this research showed a relationship between academic success and a productive mathematical identity, the practice of retaking a test for greater academic success would also yield a more productive mathematical identity. Another practice to consider is allowing students to take a pre-test whereby the pre-test grade does not count toward a course grade. Feedback on the pre-test would provide students opportunities to gauge their current mathematical understanding. This also provides students opportunities to target their studies toward understanding concepts for test preparedness and academic success.

Student preparedness and confidence in their mathematical ability or knowledge of concepts before taking a graded assessment is another aspect of consideration for grading reform. I often tell teachers that if they are using formative assessment practices to adjust their teaching and address student learning while learning is occurring, teachers should already know how students will perform on a summative test. Formative assessment practices are intended to be used often while learning is occurring, providing
information to both the teacher and students on where they are in their mathematical understanding (Marzano, 2010). Further, since learning is still occurring as formative assessments are being administered, there should not be a grade attached to them. Or, if a grade is conferred to a formative assessment, those grades should be adjusted as students show concept understanding. Grading practices suggested here are intended to focus student classroom experiences as learning opportunities, engaging students as active participates in the learning process. When students become active members of their own learning, their willingness to participate and perform increases (Turner, et.al., 2013).

**Change Structures of Ninth Grade Mathematics**

Another implication of this research is for schools to consider alternative and innovative structures to ninth grade mathematics. As noted previously in this paper, the NCTM publication Catalyzing Change (2018) articulates the need for reform, discussion and research in high school mathematics. The reform area specified in the NCTM publication that speaks to creating equitable school structures is relevant to my research and implores researchers, professional developers and school/district administrators to be innovative in their approach to reforming high school mathematics.

This research study was conducted in the school structure of tracking. Tracking is one of inequitable school structures that NCTM beseeches educators to address (NCTM, 2018). The school principal noted that she looks forward to the day that her school can de-track and start all her incoming freshman in college-going mathematics classes as soon as they step onto the high school campus as ninth graders. However, the reality is that most students are not prepared to enter ninth grade mathematics when they step on campus the first day of school. In 2019 California state test results indicate that only 37%
of eighth-grade students met or exceeded standards in mathematics at the end of the school year (CDE, 2019). These assessment results suggest that the majority of California ninth grade students are stepping onto their high school campus with gaps in mathematical understandings or skills that may impact successful student progression through A-G\textsuperscript{16} college preparatory coursework. I propose that educators need to be innovative in school structures beyond and including detracking in meeting students’ mathematical needs.

Innovativeness in the school structures of ninth grade mathematics must include not only addressing academic preparedness but must also include structures addressing student mathematical identities due to the relationship between academic success and mathematical identity. The principal of my research school asked of the professional developers to be innovative in their reform curriculum. Although there was some academic success from this reform curriculum and in this study, there is need for innovative ideas beyond curriculum reform. The following are two examples of innovative ways schools might consider restructuring students’ mathematical experiences in ninth grade.

**Mandate early start for all ninth grade students prior to the regular school year start date.** During early start weeks, students engage in STEM projects aligned to high priority 8th grade standards and standards that will be taught in the first quarter of ninth grade. By focusing on high priority eighth grade standards, students have opportunities to solidify concepts and skills in which they have already engaged and may have not yet understood. By additionally focusing on standards for the first quarter of

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\textsuperscript{16} A-G coursework is a series of high school courses students are required to complete for admission into the California public university system, UC and CSU.
ninth grade, students are provided with mathematical opportunities to engage in forthcoming concepts and skills, laying the foundation for a deeper understanding of mathematics.

This mandated early start program, should be designed to provide students with strategies for mathematical sensemaking, strategies to monitor their own learning progress, experience mathematical academic success, develop a conceptual understanding of the content and skills for the first quarter of ninth grade mathematics, engage in identity constructing activities including motivation to learn, confidence to engage in problem solving, growth mindset, mathematical curiosity and develop agency to ask questions in class.

**Mandate two simultaneous mathematics courses for all students during ninth grade freshman year.** These mandated courses are designed such that one course is an A-G course that uses equity-based teaching practices and grade-aligned mathematical standards and curriculum. The other course is a STEM-type course. The STEM-type course includes mathematical content that draws from concepts and skills that students will be learning in their A-G course in the next semester. The STEM-type course is designed to provide students with opportunities to be introduced to and engage in conceptual and application type activities that provide a mathematical foundation and scaffolds for new mathematical understandings. Additionally, the activities and tasks students engage in during the STEM course can be used as concept launching points (lesson hooks) and scaffolds of concepts when these concepts are taught in the regular A-G course. The idea of these simultaneous courses is that students are both fulfilling A-G requirements in their A-G course and at the same time the STEM-type course is
providing a foundation for the upcoming concepts students will learn and provide more opportunities for mathematical success which will support the construction of a positive mathematics identity.

Understand the Interrelated Nature of Constructing Mathematical Identity

Lastly, an implication of this research is that teachers need to understand the interrelated network of influencing factors that contribute to the construction of student mathematical identity. With support of professional developers and instructional coaches, teachers could use this network understanding advantageously toward positive change in the classroom. Since student mathematical identity has a relationship to student outcomes, the implications of addressing the interrelated network could also positively influence student mathematical outcomes.
References


Re: Expeditied - Initial - IRB-2020-6, The Construction of Student Mathematics Identity in Relation to Academic Achievement within a Reform Initiative

Dear Ann Trescott:

The Institutional Review Board has rendered the decision below for IRB-2020-6, The Construction of Student Mathematics Identity in Relation to Academic Achievement within a Reform Initiative.

Decision: Approved

Selected Category: 7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

Findings: None

Research Notes:

Internal Notes:

Note: We send IRB correspondence regarding student research to the faculty advisor, who bears the ultimate responsibility for the conduct of the research. We request that the faculty advisor share this correspondence with the student researcher.

The next deadline for submitting project proposals to the Provost’s Office for full review is N/A. You may submit a project proposal for expedited or exempt review at any time.

Sincerely,

Dr. Thomas R. Herrinton
Administrator, Institutional Review Board

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Apr 8, 2020 10:16 AM PDT

Ann Trescott
Sch of Leadership & Ed Science

Re: Modification - IRB-2020-6 The Construction of Student Mathematics Identity in Relation to Academic Achievement within a Reform Initiative

Dear Ann Trescott:

The Institutional Review Board has rendered the decision below for IRB-2020-6, The Construction of Student Mathematics Identity in Relation to Academic Achievement within a Reform Initiative.

Decision: Approved
Findings: None
Research Notes:
Internal Notes:

Note: We send IRB correspondence regarding student research to the faculty advisor, who bears the ultimate responsibility for the conduct of the research. We request that the faculty advisor share this correspondence with the student researcher.

The next deadline for submitting project proposals to the Provost's Office for full review is N/A. You may submit a project proposal for expedited or exempt review at any time.

Sincerely,

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