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**THE EFFECTS OF DIFFERENT TYPES OF PROMPTS ON ACHIEVEMENT AND
ATTITUDE IN MATHEMATICS**

by

Barbara R. Greer

A Dissertation Submitted to the Faculty of
San Diego State University and the University of San Diego
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Education

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May 2009

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ABSTRACT OF THE DISSERTATION

**The Effects of Different Types of Prompts on Achievement and
Attitude in Mathematics**

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Barbara R. Greer

Doctor of Education

San Diego State University and the University of San Diego, 2009

This study examined the effects of three different types of writing prompts, procedural, summary, and self-monitoring, on achievement and self-concept of ability in mathematics. Participants included 81 eighth grade students taking a course designed to prepare students for algebra in the ninth grade in a large urban school district in Southern California. Data were gathered using a quasi-experimental design, teacher-researcher created pre-and post-tests, the Minnesota Mathematics Attitude Inventory, teacher field notes, student responses to prompts, and individual and group interviews. Controlling for demographic and other variables identified in the study, simultaneous regression analysis revealed that only summary writing had a significant positive association at the .05 significance level on achievement and no type of prompt was associated with changes in self-concept of ability. Self-concept of ability, however, was found to have a small, positive association with achievement gain. Qualitative analysis revealed several themes, including resistance to writing, elaboration, writing as a reference, grading student writing, the inability to express thoughts when understanding is limited, and writing and remembering. Student self-reports revealed complex relationships between content, instruction, achievement, attitude, and writing. While procedural prompts were preferred by most students, all three types of prompts were found useful by students at different times during the study.

The teacher-researcher concluded that the nature of the content and the level of students' understanding should be considered when selecting the type of writing prompt to complement instruction in mathematics at any given time. Different types of prompts "fit" the content and level of students' understanding better than others. Prompts must be purposefully selected to focus students' attention on the type(s) and level of knowledge required by the curriculum. In addition, students who are struggling with understanding a concept or mastering a skill may benefit more from being able to identify and express their understanding and confusions through self-monitoring than through more informational types of writing. Instruction in and use of a variety of carefully selected prompts in mathematics may give students and teachers an effective alternative to assigning more problems without increasing teacher workload and increasing opportunities for students to gain access to the content.

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CHAPTER 1

INTRODUCTION

As a middle school mathematics teacher, I have witnessed first-hand the struggle of many students to understand and find success in mathematics. Our society tells them it is alright to have trouble with school mathematics and their parents tell them they have inherited their lack of mathematical “genes.” But the students know better. They want to understand and master mathematical skills and concepts as much as they want to be able to read. The more experienced I have become as a teacher, the more tools, such as manipulatives, problems in context, and clearly stated tasks and task expectations, I have gained to help students think mathematically.

Even with the many tools my colleagues and I have accumulated over the years, many students, especially those of subdominant cultural and language groups, continue to struggle with school mathematics as my colleagues and I continue to search for ways to help these students gain the basic skills and conceptual understanding they need to be successful in school mathematics. At the same time, middle school teachers are overwhelmed with the number of skills and concepts they must try to help students understand and master. Instructional time is at a premium and the mathematics teachers I know will only consider new strategies that directly affect students’ achievement in class and on state-mandated achievement tests. When writing is mentioned as a possible tool for improving student achievement in mathematics, they typically react against it, unwilling to invest large amounts of class time having students go through the full process of pre-writing, writing, revising, and publishing. In addition, they feel unprepared to “teach writing,” just as many literacy educators feel unprepared to teach mathematics. But the meanings of mathematical concepts are mediated through language, and as mathematics teachers we must help students understand the content of our curriculum through language. There is simply no other way. This study is an attempt to turn an already acknowledged effective practice, that of reflection, into a more effective tool for improving students’ understanding of mathematics through writing without adding significantly to teachers’ already full curriculum.

BACKGROUND TO THE STUDY

For many years, policymakers, educators, national organizations, and researchers have been looking for ways to improve mathematics education. The impetus for improvement has come from a variety of sources in recent years, including the results of the 2003 Third International Mathematics and Science Study (TIMSS), the National Assessment of Educational Progress (NAEP), and assorted state achievement tests. For example, the Third International Mathematics and Science Study (National Center for Education Statistics [NCES], 2003) has brought to light what has been reported in the media as a potentially disturbing problem in mathematics achievement in the United States. While this international study of 4th and 8th students in mathematical performance in 46 nations places 8th grade U.S. students well above the international average in mathematics and well above 26 countries, U.S. students scored well below nine other countries, including Singapore, the Republic of Korea, Hong Kong, Chinese Taipei, and Japan, all U.S. competitors in science and technology. Even though U.S. 8th graders continue to show improvement in mathematics performance from 1995 to 2003, there is a great concern that the United States is unable to compete in a growing global economy and maintain its superpower status through scientific superiority.

A more immediate, and possibly more accurate, picture of trends in student achievement in mathematics in the United States comes from the National Assessment of Educational Progress (NAEP). This series of studies, mandated by the United States Congress in 1988, measures and compares U.S. student achievement in reading, mathematics, science, writing, history, and a variety of other subjects from 1990 to the present (NCES, 2003). Student performance is measured in terms of scale scores from 0-500 and reported as Basic, Proficient, or Advanced, where Basic denotes partial mastery of skills, Proficient indicates, solid academic performance, and Advanced indicates superior academic performance. Some of the comparisons made in the NAEP include year-to-year, between grade levels, states and regions, and between cultural, linguistic, and socioeconomic groups. While the NAEP has shown steady progress in mathematics within grade levels and within groups, only 30% of the nation's 8th grade students performed at or above Proficient in 2005, indicating a need for improvement in mathematics education across the nation. Similarly, as White, African American, and Hispanic student scores continue for the most part to rise from

year to year, a persistent gap in achievement between White students and their African American and Hispanic peers is indicated by NAEP data, with only 13% of African American students and 9% of Hispanic students achieving at the Proficient or above level at 8th grade. Furthermore, only 13% of students from among the lowest socioeconomic groups, as identified by eligibility for free or reduced lunch, scored at or above proficient at 8th grade. Clearly, the NAEP data indicate a need to improve mathematics education in the United States, especially for ethnic and linguistic minorities and for those who are economically disadvantaged.

At the state level, the results are much the same as those at the national level for performance in mathematics. The California Standards Test (CST) is a criterion-referenced test that compares student scores against a predetermined standard of achievement for each curriculum standard, such as applying fractions in context or solving one-step linear equations. Tests are scored and reported in much the same way as the NAEP, with performance levels of Far Below Basic, Below Basic, Basic, Proficient, and Advanced (California Department of Education, 2006). In order to make comparisons between NAEP results and CST results, only the categories of Basic, Proficient, and Advanced are included, with categories below Basic subsumed under the Basic title. By 7th grade, only 37% of students are achieving at or above Basic in mathematics in California.

Results are even more discouraging for California's 8th graders. The 8th grade mathematics standards for California students are Algebra I standards. As such, students performing at grade level are expected to take and be successful in Algebra I. However, there are a number of possible courses and accompanying tests available to 8th grade students, depending on their previous achievement in mathematics, complicating a comparison of results for 8th grade students on the CSTs. However, these results are similar to those reported at lower grade levels in the state and at 8th grade across the nation. Among the most struggling 8th grade mathematics students, those who take the General Mathematics Tests, which address 6th and 7th grade standards, only 26% scored at or above Proficient, and of students who took Integrated Math, only 17% scored above the Basic level. Of those in Algebra I, the curriculum students are expected by the state to take at 8th grade, only 34% scored at or above Proficient. Not surprisingly, the results are somewhat better for the most advanced students, those taking Geometry, normally a 9th grade course, in the

8th grade, where 79% of students in the course scored Proficient or above. The gaps in achievement between whites and other cultural and linguistic minorities are again evident at the state level, where African American and Hispanic students score significantly below their White and East Asian peers (see Figure 1), suggesting a need for changes in current mathematics education practices.

Figure 1. Comparison of California proficiency levels by group. Source: California Department of Education (2006), *California standardized testing and reporting (STAR)*, retrieved February 28, 2006, from <http://star.cde.ca.gov/star2005/Viewreport.asp>.

Over the past several years, many reforms and improvements instruction have been suggested and explored in order to improve student achievement in mathematics, including cooperative groups (Sherman & Thomas, 1986; Slavin, 1984; Webb, Farivar, & Mastergeorge, 2002), the use of manipulatives (Fuson, 1998; Weiss, 2005), and reciprocal teaching (Taylor & Cox, 1997; vanGarderen, 2004). In 1989, the National Council of Teachers of Mathematics (NCTM) published their Curriculum and Evaluation Standards for School Mathematics in order to address the nation's growing concern for the performance of students in mathematics in an economically and technologically changing world, presenting a

vision of mathematics education in which students are actively involved in meaningful problem-solving activities with an emphasis on the social construction of knowledge and meaning-making posited by Vygotsky (Moll, 1990). The emphasis on language, discussion, and communication in the classroom, as ways to support the learning mathematical skills and concepts, is carried over into NCTM's subsequent publication, *Principles and Standards for School Mathematics* (NCTM, 2000), in which the standard of communication, in both written and oral forms, is one of the five process standards that are an integral part of a complete and effective program of mathematics instruction across all grade levels. According to the authors, "Students who have opportunities, encouragement, and support for speaking, reading, writing, and listening in mathematics reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (p. 60). Furthermore, "Because mathematics is so often conveyed in symbols, oral and written communication about mathematical ideas is not always recognized as an important part of mathematics education" (p. 60). According to Cobb, Wood, and Yackel (1994, as cited in NCTM, 2000), communicating about mathematics does not come naturally to most students, and teachers need to help them learn to develop their mathematical thinking through mathematical language. For these reasons, NCTM looked to then contemporary research and practices in both mathematics and other content-area disciplines.

NCTM's Communication principle has its roots in the Writing to Learn (WTL) movements of the 1970s, when Emig (1977) and Britton (1970) laid the theoretical foundations in place that have continued for the writing-to-learn movement. The work of Britton and many other researchers in England provided the rationale for writing and language across the curriculum, but emphasized the personal, linguistic, and psychological growth of students above that of content knowledge and the linguistic demands of the disciplines (Ackerman, 1993; Applebee, 1974). Britton also took a holistic view of language, investigating the ways in which speaking, listening, reading, and writing mediated learning. Emig, on the other hand, focused on writing and its role in the construction of knowledge. Her theories, based upon her own research on individual composing processes, emphasized three unique properties of writing among other language media: (1) Writing provides immediate feedback by which the writer/learner can modify her thinking; (2) Writing requires the establishment of systematic relationships; (3) The level of personal engagement

in the writing process is generally higher than in any other language medium. These properties comprise some of the most commonly studied attributes of writing-to-learn activities in mathematics over the past 30 years.

Many studies over the past 35 years have described both the benefits and costs of Writing to Learn (WTL) in mathematics for both teachers and students. One of the benefits to teachers was increased awareness of individual student needs, both cognitive and affective (Baker, 1994; Brown, 1995; Kasperek, 1993; Pugalee, 2004; Stewart, 1992). Additionally, writing in mathematics was found to provide teachers with a powerful and immediate tool for formative assessment leading to changes in curriculum or course design (Millican, 1994; Mower, 1996; Quinn & Wilson, 1997; Stewart, 1992). Writing was also found to be a window into student thinking, including the presence and development of metacognitive strategies (Clarke, Waywood, & Stephens, 1993; Pugalee, 1995), and to increase student engagement, ownership, and excitement about mathematics (Curcio & McNeese, 1993; Kliman & Kleiman, 1992; Millican, 1994; Scott, Williams, & Hyslip, 1992). Furthermore, teachers also reported that some students found writing to be a way for students who are reluctant to ask questions or make contributions orally in class discussions to ask their questions and express their thoughts (Mower, 1996).

Unfortunately, from the teachers' perspectives, some of the costs included the extra time needed for teachers to give feedback (Lim & Pugalee, 2005; Mower, 1996; Quinn & Wilson, 1997), and controversies surrounding how to grade writing in mathematics (Andrews, 1997; McIntosh, 1991). Teachers also reported decreased in-class time usually spent on practice and conceptual instruction as the teacher learned how to help students write, especially with low-achieving or at-risk students (Mower, 1996; Quinn & Wilson, 1997). Student resistance to writing in mathematics (Baker, 1994; Brown, 1995; Mower, 1996) was also found to be a drawback to writing in mathematics, though student resistance was generally overcome after writing became incorporated into the curriculum on a regular basis.

Overall, teachers who used WTL in mathematics in the research reported positive attitudes towards incorporating these strategies in their instruction even though they may have experienced increased demands on their time both inside and outside the classroom. Despite the costs, they recognized the positive effects WTL tasks had on students' reported

understanding of, and attitudes toward, mathematics. Two of the benefits reported by students were an increase in student comprehension and construction of personal meaning (Mower, 1996), as well as an awareness of what they did or did not know about the topic and other metacognitive information (Pugalee, 1995; Shield & Galbraith, 1998; Stewart, 1992). Students also found writing helped them organize and clarify their thoughts (Kasperek, 1993), and to make connections to real-world situations and between and within mathematical structures (Curcio & McNeese, 1993). In addition to cognitive benefits, purposeful, integrated WTL activities in mathematics also resulted in increases in positive student attitudes towards mathematics. Students reported a reduction in anxiety (Borasi & Rose, 1989; Brown, 1995), the perception of a more personal atmosphere (Holens, 1996; Kasperek, 1996), and a general improvement in student attitudes towards mathematics (Scott et al., 1992), especially among low-achieving students (Baker, 1994).

Though many qualitative, descriptive studies support the theoretical promises of writing to learn as a way to increase student achievement through enhanced engagement and knowledge construction, Ackerman's (1993) review of the literature, Bangert-Drowns, Hurley, and Wilkinson's (2004) meta-analysis of the status of writing to learn in mathematics, and various quasi-experimental studies, have reported mixed results. In their three-year research of writing in high school science and social science classrooms that included three experimental studies, Langer and Applebee (1987) concluded that, while manipulating information through writing increases understanding, the writers learned a limited band of information. In addition, they found that writing tasks differed in engagement and learning potential, and that "different kinds of writing activities lead students to focus on different kinds of information, to think about that information in different ways, and in turn to take quantitatively different kinds of knowledge away from their writing experiences" (p. 135). Additionally, writers' prior subject-matter knowledge may have had a negative effect on the potential for writing as a mode of learning for some students. Ackerman (1993) extended Langer and Applebee's findings in his review of 35 empirical studies having to do with writing across grade levels and disciplines. He concluded that, "the proof negative for why writing does not enhance learning because it draws attention to a host of complicating factors in learning and literate practices" (p. 360). Among these factors were institutional and

disciplinary practices, conflicting cultural practices among increasingly diverse student populations, subject matter, and qualities in writing and learning tasks.

In Bangert-Drowns et al.'s (2004) meta-analysis of writing to learn, the researcher examined 46 control-treatment studies across grades levels and disciplines, half of which were in mathematics. This study identified several factors related to the effectiveness of writing to learn and their relationship to student achievement gains. In 96% of the studies, the writing tasks prompted informational use of text, including descriptions and summaries of information along with personal forms of writing, and only 37% asked for metacognitive reflection. The researchers found, however, that "...studies of personal writing showed no significant relationship with effect size while prompts that encouraged student reflection about confusions, current knowledge state, and learning processes, ...proved particularly effective" (p. 12). In addition, while Bangert-Drowns et al. found a small but significant positive effect on achievement in 75% of the studies involved, results were significantly lower for those in grades 6-8, a phenomenon which could be the result of developmental issues, unidentified features of instruction at this grade level, or even the presence of more guided self-reflection in these grades, thus reducing the comparative gains between writing to learn treatment groups and control groups at this level. Furthermore, longer writing tasks were related to smaller outcomes and, contrary to theoretical predictions, feedback showed no significant relationship to effect size. Finally, the length of treatment had a significant impact on effect size, possibly indicating the positive cumulative effects of writing to learn over time. Overall, Bangert-Drowns et al. concluded that, "it is hard to equate writing with learning and thinking" (p. 13) based on the results of his review. However, the research in this review does suggest that writing need not be elaborate or take large amounts of class time, that learning from writing may be optimized by contextual factors, such as specialized instruction in disciplinary genres and metacognitive processes, especially for transitional students, such as those in grades 6-8. He does, however, caution that previous instruction in metacognitive strategies may neutralize the comparative benefits of writing to learn.

Quasi-experimental research of writing to learn in mathematics has also yielded mixed results in both achievement and attitude gains. In most of these non-randomized, control-treatment studies, the students either received instruction with writing to learn tasks or instruction without writing tasks. Academic gains were assessed mainly via teacher-made

or content-related materials (Baker, 1994; Giovinazzo, 1996; Jurdak & Zein, 1999; Kasperek, 1993; Millican, 1994; Porter & Masingila, 2000; Resnik, 1992), though one compared pre-and post-treatment standardized achievement test scores (Brown, 1995), and one a combination of the two (Henn, 1989). Quantitative methods generally found no significant difference between control and treatment groups (Baker, 1994; Brown, 1995; Giovinazzo, 1996; Jurdak & Zein, 1999; Porter & Masingila, 2000), though a few found some significant difference between groups on some measures (Kasperek, 1993; Millican, 1994; Resnik, 1992) and Millican (1994) found a significant difference in achievement gains for low achieving fourth grade students who engaged in writing to learn tasks in mathematics.

As mentioned above, one of the moderating variables thought to be predictive of student achievement in mathematics and tested in several studies of writing to learn in mathematics is student attitude towards mathematics. While some studies have found that attitude towards mathematics had a significant positive effect on achievement, quantitative results in general have not overwhelmingly supported this theory or report that attitude may have had an indirect positive effect rather than a direct effect (Hammouri, 2004). In their research aimed at determining the causal relationship between attitude toward mathematics and achievement in mathematics, Ma and Xu (2004) confirmed previous research that found prior student achievement in mathematics, “demonstrated causal predominance over attitude across the entire secondary school” (p. 256), with high achievement (elite status) as a moderating variable. McCoy (2005) also found attitude, measured by the sum of the scores on an instrument that includes subtests of confidence in using mathematics, perceived usefulness of mathematics, and math anxiety in its construct of attitude toward mathematics, had no significant effects on achievement as measured by state achievement tests, though attitude did have a significant effect on student grades in algebra. In fact, post-attitude scores were significantly lower at the end of the study for each of the subtests included in the attitude measurement instrument. This decline in attitudes towards mathematics, specifically students’ self-concept of ability in mathematics, during the transitional grades of middle school was also identified by Eccles et al. (1989), a phenomenon deserving of further study in order to find effective interventions to prevent such a decline.

The studies mentioned above point out one of the main difficulties found throughout the literature that attempts to determine the effects of attitude toward mathematics on student

achievement in mathematics: there is no generally accepted definition of attitude or standardized measurement instruments, though a few occur more or less frequently in the literature. Comprised of many subconstructs, attitude is a concept that may include not only those identified by McCoy (2005), but may also include perception of the teacher, enjoyment in mathematics, motivation, epistemological beliefs about mathematics, perception of mathematics materials, and many other motivational, perceptual, and personality. In addition, the researcher may choose to sum the scores of an instrument and call the variable simply “attitude toward mathematics,” while others focus on the effects of one or more of the individual subconstructs on achievement, making it difficult to draw any conclusions about the current state of the evidence with regard to the effect of attitude on student learning in mathematics.

One subconstruct of attitude or motivation, self-concept of ability, shows some promise as a predictor of achievement in mathematics. Related to self-confidence and perceived self-efficacy, current theory is founded on that of Bandura (1986), who identified the relationship between global self-conceptions and perceptions of self-efficacy in particular situations. Specifically, composite self-images are not necessarily good predictors of how people may behave in a situation in which they believe they are not competent. Students who have high global self-images may still perceive themselves to be less capable in specific content areas, such as mathematics, science, or history. As early as 1964, studies showed a significant and positive correlation between self-concept of ability in specific subject areas and achievement in those areas, though no attempt was made to determine causal relationships (Brookover, Thomas, & Paterson, 1964).

In addition to differentiation between global and specific self-concepts of ability, social cognitive theory also points out that people are more likely to choose to persevere and expend effort in areas in which they perceive themselves to be the most able. It follows logically that students are more likely to make achievement gains in subjects in which they choose to expend the effort necessary to address and complete assigned tasks. In addition, achievement has been shown to be a cause of self-concept enhancement (Helmke & vanAken, 1995). In other words, students who achieve more in a given subject perceive themselves as more efficacious, leading to even higher achievement in that subject through increased effort and perseverance. It is important, then, to find ways to build students’

self-concepts of ability in order to increase student achievement in specific subject areas, such as mathematics. Writing to learn may be one of those ways.

PROBLEM STATEMENT

While many studies have been conducted to describe the effects of writing to learn tasks in mathematics, including the benefits and drawbacks, and to examine the relationship between writing and non-writing treatments and their effects on student achievement in mathematics and attitude towards mathematics, the results remain mixed. In addition, though studies have described and categorized the types of responses students produced when engaging in writing to learn tasks (Clark et al., 1993), few have focused on the actual prompts used or the effects of these prompts on achievement or attitude (Miller & England, 1989), and none has compared the efficacy of one type of task as defined by its prompt with another. Furthermore, few studies have examined the effects of writing to learn tasks in mathematics on the achievement of struggling middle school students and their self-concepts of ability.

PURPOSE OF THE STUDY

The purpose of this study is to examine the relationship between the kinds of questions asked by the teacher and student performance in mathematics. Since the nature of the prompt defines the nature of the writing task, the words prompt and task will be used interchangeably throughout this study. The study will examine whether writing prompts that focus on higher level thinking, such as comparison, self-monitoring, and summarization, will result in higher student performance on teacher-created assessments than those that require lower level thinking, such as simple explanations and descriptions of procedures and concepts. In addition, the study will examine the effects of various writing to learn tasks on student self-concepts of ability.

The study will be conducted using mixed methods. Utilizing a quasi-experimental, non-randomized design, three classes of the lowest 8th grade students will make up three different treatment groups. All classes will receive the same content instruction. Only the type of prompt and the discussion surrounding ideas in response to the prompt will differ, except where instruction naturally changes as a result of student participation throughout the lesson. One class will receive instruction in answering and responding to procedural prompts,

the lowest level of knowledge in this experiment. Another class will receive instruction in and respond to summarization prompts, and the third will receive instruction in and respond to metacognitive prompts, specifically self-monitoring tasks. For ethical reasons, the classes will rotate at least once through the different types of prompts in order to ensure that students receive equitable instruction.

In addition, the study will examine the relationship between student attitude and writing through student and teacher self-report and interviews. The students will also be given the Mathematics Attitude Inventory with self-concept of ability as one subtest, at the beginning and end of the study in order to detect any measurable effects on attitude towards mathematics as a result of instruction that includes writing in mathematics.

RESEARCH QUESTIONS

This study is driven by the following research questions:

- Does the type of writing task, as defined by the nature of the prompt given, have an effect on student achievement in mathematics for low achieving 8th grade students?
- Does the type of writing task, as defined by the nature of the prompt given, have an effect on student self-concept of ability for low achieving 8th grade students?
- Does student self-concept of ability have an effect on student achievement for low achieving 8th grade students?

CHAPTER 2

LITERATURE REVIEW

This study is concerned with the effect of writing to learn tasks in mathematics on student achievement and self-concept of ability in mathematics. Therefore, this literature review includes theory and research on writing-to-learn, writing-to-learn in mathematics, prompts, or writing tasks, associated with specific types of knowledge, metacognition, and self-concept of ability.

WRITING TO LEARN

The contemporary writing to learn movement has its theoretical roots in the sociocognitive theories of Bruner, Vygotsky, and Luria and Yudovich. Their work set the foundation for understanding learning as a dynamic social process, a process in which learners must actively engage the curriculum through language and interaction with others, as well as the role of written language in support of higher cognitive functions (Bruner, 1971; Luria & Yudovich, 1971; Vygotsky, 1962).

Historically, the foundations of the writing to learn movement were set in the 1970s, as rhetoric and composition specialists in higher education began to review the traditional paradigm of writing instruction separated from disciplinary contexts (Maimon, 1982; Russell, 1990). In light of input from colleagues across the disciplines who expressed dissatisfaction with their own experiences in writing instruction as students, and their reluctance to engage in writing activities with their own students as a result, language arts instructors began to shift the focus of writing away from product, with its emphasis on correctness, usage, and grammar, and toward the process of writing, with an emphasis on how learners write and how writing can help students learn in a variety of academic settings. Language and learning came to be seen as intricately connected and integral to instruction in all academic disciplines. Eventually, language instruction in all disciplines began to be seen as necessary to provide access to discourses of power in higher education and in society in general (Russell, 1990). Called Writing Across the Curriculum (WAC), this movement placed

emphasis on how writing in the disciplines could improve student writing. However, the paradigm shift from learning to write to writing to learn was already under way. Over the next 20 years, several landmark studies and subsequent reflections on the findings of these studies would set the theoretical and empirical foundations in place that have supported the writing to learn movement. This review will focus on three of the most influential studies of the time: Britton, Burgess, Martin, McLeod, and Rosen (1975), Emig (1971), and Langer and Applebee (1987).

The work of Britton et al. (1975) provided the rationale for writing and language across the curriculum, but emphasized the personal, linguistic, and psychological growth of students above that of content knowledge and the linguistic demands of the disciplines (Ackerman, 1993; Applebee, 1974). In their study of the writing processes of 11-18 year-olds in Great Britain, Britton and his colleagues took a holistic view of language, investigating the ways in which speaking, listening, reading, and writing mediated learning. Their most influential contribution to the writing to learn movement included their categorization of student writing as transactional, expressive, and poetic, as well as several subcategories, and the relationship of the role of audience to each category. The categories, however, are not discrete. Writing may fall anywhere on a continuum between transactional at one end, through expressive, to poetic on the other end. On the poetic end of the spectrum, the writer is expressing feelings and beliefs and the content may be highly personal. Little thought is given to communicating to a specific audience, and the writer communicates through complex and subtle structural forms. In transactional writing, the writer is seeking to communicate to an outside audience or to “meet the demands of some kind of participation in the world’s affairs” (p. 83). The function of transactional writing is often to inform and its content, therefore, is more explicit, detailed, and organized in order to communicate the writer’s knowledge and ideas to a specific audience. While many subsequent studies have focused on more expressive forms, such as journal writing and the role of affect in learning, the studies of Britton and his colleagues provide a foundation for more cognitive aspects of writing to learn. Their work supports the idea that different disciplines often require particular forms of rhetoric in their writing. For instance, scientific writing was frequently informative and analogic in the earlier years and moved toward more abstract forms of speculation and tautology later in the students’ schooling.

In her detailed descriptive study of the composing process of twelfth grade students, Emig (1971) examined not only the individual composing process of high school seniors, but also the entire context surrounding the process. She brought to the fore the need for educators to focus students on the process of writing, not just the mechanics, by focusing on writing and its role in the construction of knowledge. One of her most influential contributions to the discussion of writing across the curriculum was her view of writing as a unique mode of learning: "Writing serves learning uniquely because writing as process-and-product possesses a cluster of attributes that correspond uniquely to certain powerful learning strategies" (Emig, 1977, p. 89). Citing Bruner's (1971) categories of the three major ways in which we learn--enactive (doing), iconic (imaging), and representational (symbolic)--Emig makes the case that writing engages the learner in all three ways simultaneously, or nearly simultaneously. For Emig, when writing, "...the symbolic transformation of experience through the specific symbol system of verbal language is shaped into an icon (the graphic product) by the enactive hand (Bruner, 1971, pp. 7-8). In addition, Emig's view of writing as a unique mode of learning is supported by Vygotsky (1962), who noted that, when writing, learners must engage in "deliberate semantics", or "deliberate structuring of the web of meaning" (p. 100) as part of the process of Bruner's imaging. Furthermore, citing Luria and Yudovich (1971), Emig recognizes the necessity in writing to slow down thought processes, thus encouraging learners to develop their thoughts by moving between past, present, and future experiences, and recursively analyze and synthesize. Finally, Emig emphasizes the epigenetic nature of writing, its visibility and availability to the writer throughout the writing process. As a concrete form of immediate feedback, writing provides the learner with a record of their own evolutionary thought process, something they can edit and reformulate through analysis and synthesis. In much of the writing to learn literature in mathematics, the unique attributes of writing are summarized as follows: (1) Writing provides immediate feedback by which the writer/learner can modify her thinking; (2) Writing requires the establishment of systematic relationships; (3) The level of personal engagement in the writing process is generally higher than in any other language medium. These properties imply that writing is more than just a means of communication; it is also closely related to thinking and a powerful aid to learning content. Those same properties comprise some of the

most commonly studied attributes of writing-to-learn activities in mathematics over the past 30 years.

The link between writing and learning was further explored in American schools in the mid-1980's by Langer and Applebee (1987). Their work was a response to the findings of the 1981 National Assessment of Educational Progress, which found that, while American students could read a range of material and formulate in writing a superficial response to what they read, they could not adequately analyze information, justify their points of view, or otherwise think critically about what they had read or written. Their view of the role of writing in thinking, especially Britton's transactional mode of writing, echoes that of Emig. They conceptualized that role as a combination of the permanence of the written word (immediate, concrete feedback), the explicitness required in writing if it to retain its meaning in a variety of contexts, organizing thoughts and thinking through relationships between ideas (the explication of systematic relationships), and the active nature of writing (personal engagement). Their study of writing in high school content area courses examined the effects of different types of writing tasks on learning and on the implementation of writing activities to support instruction in the classroom. Using both descriptive and inferential statistics, as well as detailed descriptions of observational data and other qualitative methods, they found that, across disciplines, the presence of writing activities did assist learning and that different kinds of writing activities led students, "to focus on different kinds of information, to think about that information in different ways, and in turn to take qualitatively and quantitatively different kinds of knowledge away from their writing experiences" (p. 135). For instance, short-answer study questions led to short-term recall of much specific information while more analytic writing led to, "a more thoughtful focus on a smaller amount of information" (p. 135), and that this smaller amount of information may be remembered for a longer time. Summary writing also led students to focus more on the whole text, but in more superficial ways than in deep analytic thinking.

In addition to exploring the effect of writing on student learning, Langer and Applebee (1987) also examined instructional practice around writing in subject areas. They found three main ways in which subject-area writing could be used productively: (1) to assess, access, and activate prior knowledge; (2) to review and consolidate what has been learned; (3) to reformulate and extend ideas and experiences. The most frequently used form

of writing in the subject areas was review writing. While review writing can be used to help students rethink and clarify new learning, the researchers found that most teachers used review writing to evaluate students' learning. Only when teachers changed their evaluation of student writing from a focus on recitation to the quality of their thinking processes did the writing activities become more effective tools for learning. The authors support their model of instruction citing, as did Emig, the work of Bruner and Vygotsky. To be an effective instructional practice, according to the researchers, writing instruction must provide "carefully structured support or *scaffolding* as students undertake new and more difficult tasks" (p. 139). In the process of completing writing tasks, students are given the opportunity to internalize information and strategies and to learn skills and concepts they need in order to complete more difficult tasks on their own. According to Bruner and others, language provides the basis for concept formation and is a powerful tool for cognitive growth and essential for thinking (Bruner 1966, as cited in Langer & Applebee, 1987; Bruner, 1971). Langer and Applebee's study, along with Bruner's theories and Vygotsky's Zone of Proximal Development, where communication between the learner and a more competent peer enables the learner to develop conceptual understanding, provides both empirical and theoretical support for the incorporation of writing to learn activities in subject area courses.

While the three studies above are the foundations of a large body of literature in support of writing to learn activities in content area classes, several studies have not been as positive. Ackerman's (1993) review of writing to learn studies found that empirical research of writing to learn had been mixed. He concluded, however, that these mixed results may be due to "a host of complicating factors in learning and literate practices" (p. 360). Ackerman names academic, cultural, and literacy contexts and ecologies as a few of these complicating factors and urges future researchers to consider, describe, and explore the contexts in which writing to learn activities take place when designing their studies.

In his meta-analysis of the effects of writing to learn interventions on academic achievement, Bangert-Drowns et al. (2004) found similar mixed results that Ackerman had found ten years earlier, though, overall, 36 out of 48 studies outcomes were positive. Overall, the effect was, however, "rather small." Bangert-Drowns et al., however, did explore and report on some of the academic complicating factors that affect instructional practices. For instance, in grades 6-8, the average effect size was significantly lower than the outcomes in

other grades, and four out six studies actually had negative outcomes. The researcher suggests that the unique position of the middle grades as the beginning of subject matter differentiation may play an important role in the selection of effective writing to learn activities in these grades. He specifically suggests that, “Transitioning into new subject-specific writing forms may interfere with the relationship between learning and writing” (p. 50). The author further speculates that developmental issues or unidentified features of instruction at this level may interfere with the impact of writing to learn. He even postulates that specific instruction in self-reflection in the middle grades may “diminish the comparative metacognitive enhancements offered in writing to learn activities” (p. 50). Bangert-Drowns et al. found that specific kinds of prompts were especially effective, especially those that asked students to reflect on their current knowledge, confusions, and learning processes, while personal writing showed no significant relationship to effect size. This finding supports Langer and Applebee’s (1987) conclusion that writing was an effective tool in scaffolding students’ metacognitive and self-regulatory processes. However, another finding in Bangert-Drowns et al.’s study conflicts with the findings of Langer and Applebee. Bangert-Drowns et al. found that shorter writing assignments actually had a greater effect on learning than longer assignments, possibly due to a decrease in motivation on longer assignments or to the superficiality of the assignments themselves. These findings emphasize the importance of Ackerman’s conclusion that the effectiveness of writing assignments depends on the entire context of the classroom and culture and other complicating factors. Interestingly, Bangert-Drowns et al. also found that, while long-term studies were generally associated with higher effect sizes, though they did not achieve the study’s significance criterion of $p < .05$, rendering the importance of treatment length inconclusive.

Finally, Ochsner and Fowler’s (2004) points out several problems in the writing to learn literature. The authors, like Ackerman, point out that the case for writing to learn (what they refer to as Writing in the Disciplines, or WID) should ultimately rest on actual improvement in academic performance. Instead, the case for student achievement “remains more asserted than achieved” (p. 128), and experimental studies in the field show that “writing does not dependably promote learning” (p. 128). The authors point out that the tendency in studies that show little or no positive relationship between writing and learning has been to fault the research methodology. In some studies, they found that researchers carry

a basic presupposition that writing to learn is a positive experience, even to the point of leaving the majority of studies that show null or negative results out of their reviews of literature. This tendency to assume that writing to learn activities themselves promote learning leads researchers to ignore possible “complicating factors” in the writing to learn-student achievement relationship. Ochsner and Fowler suggest that alternative interpretations of the data may lead to questioning the basic theoretical assumptions about writing as a unique mode of learning. Like Langer and Applebee (1987), they recommend asking questions about how writing interacts with other modes of learning. As a tool for learning, writing may be “necessary but not sufficient to produce higher-level thinking” (p. 128).

WRITING TO LEARN IN MATHEMATICS

For the most part, the studies in the previous section examined writing to learn in a variety of subject areas. In this section, the focus will be on the literature surrounding writing to learn in mathematics over the past 20 years.

One of the earliest studies often cited in subsequent research was that of Rose (1989). Her study is representative of a major portion of the early literature on writing in mathematics for four reasons: (1) her focus on higher-level education; (2) her focus on journal writing (Britton’s expressive writing mode); (3) her qualitative approach to the data; (4) the findings of the study. In her exploratory study of writing in a college level calculus course, Rose sought to

examine the characteristics of expressive writing that result when students are given the opportunity and incentive to write, the students’ assessments of how the writing affects their learning of mathematics, and teacher/researcher’s attempt to make meaning of the experience from her perspective and the rich data generated from the project. (p. 9)

Within the journal format, students were given many opportunities to engage in several types of writing, including free writing and prompted writing. The prompts were generated by both the teacher and the students. In free writing, students are given the opportunity to choose to write about their experiences with content, instruction, course or class management. Students may write about their feelings (affective) or about their understanding of the content (cognitive). Prompted writing asks students to respond to specific questions about student experiences and may be either affectively or cognitively oriented.

One of the major findings of many of the exploratory studies on writing to learn conducted in mathematics were the reported benefits by both students and teachers. Rose (1989) reported many such benefits, including an increase in student-teacher interactions and the empowering of students as they were given the opportunity to express their feelings about the course and its content and management. Students were able to “sound off” about their confusions and frustrations and expressing their feelings through writing had a calming effect. Students then felt more relaxed with the subject matter and less under pressure and the teacher became more aware of student concerns. They reported that writing encouraged them in their mathematical pursuits, increasing their participation in class. As Bruner theorized, students were more actively engaged, could more easily slow down their thought process for reflection, and were better able to make connections between past and present mathematical experiences. Subsequent studies also reported these and other affective benefits of writing to learn in mathematics, including increases in student engagement, ownership, and excitement about mathematics (Curcio & McNeese, 1993; Kliman & Kleiman, 1992; Millican, 1994; Scott et al., 1992), and reduced levels of anxiety (Brown, 1995). Along with Rose, later researchers reported that students perceived a more personal atmosphere (Holens, 1996; Kasperek, 1993), and that there was a general improvement in student attitudes towards mathematics (Scott et al., 1992), especially among low-achieving students (Baker, 1994).

Rose (1989) also found cognitive benefits to writing in her classes. She found that writing provided her with an effective tool for formative assessment that led to changes in instruction and design of the course, both short-term and long-term, which led to increased student understanding of the course content. Again subsequent researchers identified the same kinds of benefits, both cognitive and affective, in their own studies (Millican, 1994; Mower, 1996; Quinn & Wilson, 1997; Stewart, 1992). Students in several studies reported increase in student comprehension and construction of personal meaning (Mower, 1996), found that writing helped them organize and clarify their thoughts (Kasperek, 1993), and that writing helped them make connections to real-world situations and between and within mathematical structures (Curcio & McNeese, 1993).

Rose (1989) found herself more sensitized to student needs and able to respond in more diverse and individualized ways to her students. This sensitivity to student needs had both cognitive and affective benefits, as students became aware of her knowledge of and

response to them as individual learners and became more willing both orally and in writing to express their feelings, understandings, and confusions about mathematics, a finding reported in other research (Baker, 1994; Baxter, Woodward, & Olson, 2005; Brown, 1995; Kasperek, 1993; Pugalee, 2004; Stewart, 1992). Student writing became a window into their thinking. Later research would also identify writing as a way to detect and describe the presence and development of metacognitive strategies (Clarke et al., 1993; Pugalee, 1995; Shield & Galbraith, 1998; Stewart, 1992), and as a way for students who are reluctant to ask questions or make contributions orally in class discussions to ask their questions and express their thoughts (Mower, 1996).

While not all reported in Rose, many studies of writing in mathematics found costs as well as benefits to writing. Some of the costs included the extra time needed for teachers to give feedback (Lim & Pugalee, 2005; Mower, 1996; Quinn & Wilson, 1997), grading of writing (Andrews, 1997; McIntosh, 1991), and decreased in-class time usually spent on practice and conceptual instruction as the teacher learned how to help students write, especially with low-achieving or at-risk students (Mower, 1996; Quinn & Wilson, 1997).

One of the most interesting findings in Rose's (1989) study, and one relevant to this study, was that, without guidance and specific prompting, students tended to write more in the expressive mode than in the transactional mode. Students seldom wrote about content during free writing activities, preferring instead to write about their study habits and feelings about math. It was not until the teacher/researcher encouraged them and gave them prompts that specifically called for engagement with the subject matter that students began to engage in true writing to learn activities, at least from a cognitive perspective. When students did write about content, they said that "writing about subject matter promoted understanding, facilitated reasoning and problem solving, helped integrate the material, and reinforced learning" (p. 331).

It should be noted here that Rose (1989) did not believe that quantifying the effects of writing on student achievement was inappropriate. While the descriptive, exploratory studies recommended by Ackerman have much to contribute to the body of literature surrounding writing in mathematics, the remainder of the studies included in this review will contain at least some attention to the quantitative measurement of the effect of writing on student achievement in mathematics. As Ochsner and Fowler (2004) stated, the case for writing to

learn must rest on empirical evidence that writing to learn activities have a measurable effect on academic performance.

One of the earliest quantitative studies, conducted during the same time period as Rose (1989), examined the effect of journal writing among pre-service elementary school teachers (Henn, 1989). Like Rose, Henn chose journals as the format for her students' writing and measured student achievement through teacher-created pre-and post-problem-solving tests, as well as teacher-created content examinations. In writing activities, students responded to a variety of prompts about mathematical concepts, attitudes, goal setting, and reflective prompts about previous work. Interestingly, Henn is an example of those reported in Ochsner and Fowler (2004) who held a strong basic assumption that writing improves learning going into the study, even though there is a lack of empirical evidence to support that assumption. She stated, "We don't know to date of any studies...which prove conclusively that writing improved learning--we are sure that it does, but we're not sure it's been proven" (p. 28). In her findings, however, there was no significant difference between writing and non-writing groups, though those in the experimental did have higher mean scores on all content exams except one after controlling for writing ability. Henn questioned at the end, however, whether the small, positive results of the experimental treatments in her study were due more to the ability of the instructor to use written responses as formative assessments and to adapt instruction to the needs of the students than to the writing itself as a unique tool for enhancing cognition. In another study of writing in calculus (Porter & Masingila, 2000), the researchers came to a similar conclusion that the performance of their experimental group did not significantly differ from that of the control group because the selection of tasks that involved collaboration and other activities that encouraged student discussion and participation in the control group mediated any differences that might have occurred due to writing. While Henn's questions support those who argue that instructional context must be considered when measuring the effects of writing on student achievement, Porter and Masingila's called into question Emig's theory of writing as a unique mode of language in its ability to improve student achievement.

Henn's (1989) study is one of only a few that controlled for writing ability in their analysis of the data. Comparisons were made between below average and above average writers. Below average writers did show some positive results. They scored higher than

average and above average writers on the last exam, though again not significantly higher, and they improved with each assessment. In addition, all below average writers also improved problem solving ability, while only 26% of the control group improved in problem solving over time, and only half the above average writers' problem solving abilities showed improvement. Qualitative data from the writing ability groups indicated that above average writers were more likely to consider writing as a way to help them understand concepts and work out problems and more likely to express feelings than average or below average students while average writers' opinions about writing in mathematics were mixed and below average writers saw writing primarily as a way to ask questions without speaking out in class. Though the effects of prior writing ability were not significant in this study, Johnson (1997) did find that general writing ability had a significant main effect on writing fluency in mathematics in response to domain-free and procedural prompts. Since McCutchen (1986) asserted that children with more knowledge about a topic wrote more fluently than those with less knowledge, Johnson speculated that prior writing ability may have some effect on learning when writing is made an integral part of the curriculum in mathematics. For this reason, prior writing ability will be one of the control variables in the design of this study.

In 1992, Resnik conducted a study of fifth grade students, again using journal writing as the medium. In this case, student achievement was measured through standardized test scores and chapter tests provided by the textbook publisher, both in multiple-choice formats. The six prompts used were general and open-ended and student responses could be either cognitive or affective in nature. For instance, students were asked to write about what they learned in math that day or why they could not work an assigned problem. In the discussion section of her study, the author suggested that the prompts may actually have "stifled" the learning to write process, becoming more of a fill in the blank activity than an opportunity to reflect and elaborate on their thinking processes. As in the Henn (1989) study, there was no significant difference in achievement, controlling for gender, between the mean scores of the experimental group and the control group, except on one chapter test. The author, as described by Ochsner and Fowler (2004), attributed the lack of positive results to problems in the implementation of the treatment in the study and to the curriculum itself. One of the major problems in the implementation of the study was that the students were largely unwilling to engage in writing activities in mathematics. Student resistance to writing in

mathematics has been found to be a drawback to writing in mathematics in several studies (Baker, 1994; Brown, 1995; Mower, 1996, Rose, 1989), though student resistance was generally overcome after writing became incorporated into the curriculum on a regular basis. While participants in the present study may express a reluctance to write in mathematics, the communications standards of the NCTM have influenced curriculum in most urban areas so that writing in some form or another has become an expected part of the mathematics curriculum at all grade levels. It has been the experience of the primary researcher in the present study that students are expected to write in mathematics both at the district and the site level.

Another study of journal writing was conducted by Stewart (1992). The design of this study of writing in high school algebra classes was largely qualitative, but did have a quantitative component. Many studies in writing in mathematics utilize a mixed methodology. The two types of prompts used in this study were curriculum prompts and free-writing prompts. The purposes of and responses to the free-writing prompts reflected those suggested by Ackerman (1993) and resembled those of Rose (1989). Students were asked to respond to the whole context of the learning environment, what the researcher called "student interaction with the milieu of the school." Student responses reflected many of those found in previous and subsequent studies of this type. The teacher felt more in touch with students' needs and adjusted instruction and classroom management accordingly, though the curriculum itself was set by state requirements. The students and the teacher felt there was a more personalized, less stressful atmosphere and that instruction was more individualized. On the cognitive side, some prompts asked students to focus their writing and thinking on concepts, skills, and procedures, and were often given for the purpose of review or evaluation. Again, as found in many previous and subsequent studies, no significant differences in achievement levels were found for journal writing and non-journal-writing groups on pre-and post-test scores. However, there was a significant difference in the gains made by the journal-writing group than by the non-writing group according to independent t-tests. In this study, anxiety rather than attitude towards mathematics was paired with achievement as dependent variables. As in many other studies, there was no significant difference between groups as measured by pre-and post-test surveys, though there the decrease in anxiety over time approached significance for the experimental group.

As in many studies on writing to learn in mathematics, Kasperek (1993) investigated both affective and cognitive effects of writing in the mathematics classroom. In her study of tenth and eleventh grade algebra students in a private school setting, participants responded to a variety of prompts that required that students engage in both expressive and transactional writing. Achievement was measured before the treatment, in the middle of the treatment, and after the treatment by drawing from multiple-choice questions from a standardized, norm-referenced test used by the school site in the past. Free-response chapter tests were also administered. These questions were presumably teacher-created, though their origin was not specifically stated in the text of the study. As in many previous and subsequent studies, the results were mixed, with significant positive results on some measures of achievement and no significant effect on other measures. Even though there was insufficient evidence to support her hypotheses that the writing group would show significant gains in achievement, she speculated from patterns in the data where the differences in the means between the groups increased over time that in a longer study, participants in the experimental group may have performed significantly higher than those in the control group. She concluded from the trend she saw in the data that there may, indeed, have been a cumulative effect on the mathematical achievement of the experimental group. However, these patterns were not consistent and other reasons for the increases in the differences between the means were also suggested in her conclusions. Two important reasons were given for this phenomenon and warrant consideration here. First, the teacher-created chapter tests, where the experimental group performed significantly higher on three out of the five tests, were more closely aligned with the material the students were learning than those drawn from standardized tests, a problem found in later research (Brown, 1995), and taken into consideration in the present study. This supports previous research by Hynd, Simpson, and Chase (1990) that indicated that the effects of journal writing are more visible if they are closely correlated to a criterion task. Second, the chapter tests required students show their work. By showing their work, many students may have paid more attention to their procedural work, increasing the likelihood of a correct answer. In addition, the short response questions asked students to explain their work, a factor that favored the experimental group since participants in the control group were not used to explaining their procedures. By mixing measures of achievement, with one

type perhaps biased toward the experimental group and the other the control group, the results were somewhat difficult to interpret.

Recognizing the possible importance of the relationship between students' attitudes towards mathematics and their performance in mathematics, this study represents the many studies that also included a pre-and post-survey intended to measure students' attitudes towards mathematics before and after writing treatments. As in many of the studies that included attitude towards mathematics in their design, Kasperek (1993) found no significant difference between students' attitudes towards mathematics on pre-and post-measures. One of the reasons for this finding suggested by the researcher was that the sample consisted of mostly highly motivated private school students who were used to a curriculum that called for a lot of writing in all subject areas. This is consistent with Bangert-Drowns et al.'s (2004) theory that specific features of middle school curricula in which students are taught to engage in a variety of self-reflection activities might diminish the effects of writing to learn activities. The same effect was found in a later study of journal (Jurdak & Zein, 1999), who also included writing ability as a moderator variable and attitude as a dependent variable. The middle school from which the sample was drawn was located at an international college, where students' parents were likely to be highly educated and the students more highly motivated than those in an American public school.

Jurdak and Zein's (1999) work was one of few studies that did find a significant relationship between writing and non-writing groups in conceptual understanding, procedural knowledge, and, especially in mathematics communication. However, no significant effect was found controlling for any of their moderator variables and no significant difference in either problem-solving, school-mathematics (as measured by the mean scores on school tests), or attitude was found between experimental and control groups. This is interesting in that, presumably, the school tests would have been the most closely aligned to what the students were studying. Perhaps, as in Kasperek, there was a bias toward the writing group in the areas of procedural knowledge and conceptual understanding. It seems almost certain that there would be such a bias in the area of mathematics communication. The positive findings of Jurdak and Zein are evidence of the strong link between language and concepts and the role writing played in forging a concrete link between the two. What if, as in Porter and Masingila (2000) the control group had engaged in oral language-rich activities? Would the

results have been the same? In addition, Jurdak and Zein, citing Hiebert and LeFevre (1986), attributed the effect of journal writing on procedural knowledge to the underlying effect on conceptual understanding. Since conceptual understanding leads to a deeper understanding of symbols and what they represent, this understanding leads, in turn, to more effective use of procedures. Jurdak and Zein also found no significant effect on problem-solving.

Interestingly, the researchers attributed this lack of effect on the type of prompts they used in their study. While both affectively and cognitively oriented prompts were used, they were not what the authors considered expository prompts, which they described as those that specifically require students to analyze a problem and explain their solution process. All of these reasons for the inconsistent results when testing for the effects of writing in mathematics points to the need for researchers to carefully consider and align the kinds of prompts they use, the types of knowledge on which they ask students to focus, and the types of questions asked on the measurement tools. The topics of types of prompts and types of knowledge will be addressed later in this literature review.

Another quantitative study that studied the effects of writing to learn activities on student achievement and attitude towards mathematics was that of Millican (1994). In her study of fourth graders, Millican recognized the differences between Britton's types of writing, transactional (informational), expressive (expressing feelings towards their mathematical and instructional experiences), and poetic (creating word problems), and each type of writing was included in the treatment. Millican's was one of the few studies that found a significant difference in achievement between those who engaged in writing to learn activities in mathematics and those who did not. Controlling for prior achievement, she found a significant difference between the mean scores of the experimental group and the control group on a standardized, state-mandated measure of mathematics achievement. In addition, Millican found a significant difference between the mean achievement scores of females who used writing to learn activities and females who did not and between low-achieving students in the experimental group and low-achieving students in the control group. While writing to learn activities were related to achievement in this study, no relationship was found between attitude and writing to learn activities for any group. The researcher speculated that, due to the high-stakes nature of the test used to measure achievement in this study, the teachers in the study placed far more emphasis on achievement than attitude in their instruction.

Furthermore, students were far less interested in taking the attitude post-test after the tensions produced by achievement testing than they had been in taking the pre-test at the beginning of the year.

Millican's (1994) positive findings for low-achieving students were not supported by another study of college algebra students from the same year (Baker, 1994), and no significant differences in achievement or attitude overall were found between writing and non-writing groups. However, low-achieving students did score significantly higher on the posttest attitude measurement than high achieving students. In fact, non-writing high achievers actually scored higher than the writing high achievers on the attitude posttest. The researcher speculated that writing may have provided low achieving students with a learning technique that empowered them to make connections and explore mathematical relationships in way that they were more comfortable with than traditional symbolic manipulation.

Another interesting finding was that students in the writing group who had not recently taken a mathematics course scored significantly higher on the posttest achievement measure than students in the control group who had not taken recently taken a mathematics course while control group students who had recently taken a mathematics class outscored, though not significantly, those in the writing group who had recently taken a mathematics course. Taken together with the data about high achievers, who are more likely to take more mathematics courses, and writing to learn, it seems that the findings of this study support a relationship between writing to learn activities and prior achievement in mathematics, though the nature of the relationship is still unclear.

In Baker's (1994) study, the prompts were all transactional, asking students to explain how to solve a problem, why a procedure worked the way it did, or to generalize a rule from an explanation or evidence. Several important instructional strategies were used to attempt to keep students focused on the specific task. For instance, students were reminded of the importance of the writing process as a tool for clarifying thinking and examining understanding, making clear the type of response desired. Also, the type of informational writing required was modeled for students as the instructor wrote along with students, guiding and scaffolding the process. Another important component of this study was the use of feedback. Though the role of feedback in has been examined in several studies (Brown,

1995), there is no conclusive evidence found by this researcher that it plays a moderating role between writing to learn activities and student achievement in mathematics.

In summary, the research on writing to learn in mathematics has found many benefits, including effective formative assessment and increased student engagement, as well as negative aspects, such as increased demands on teacher time and student resistance to writing in mathematics. In addition, the relationship between writing to learn and student achievement in mathematics is unclear, even after 20 years of research. The evidence for a positive relationship is mixed due to many confounding factors, including grade level, alignment in measurements of achievement, format and content of measurement instruments, differences in instruction in studies that involve multiple instructors (Baker, 1994), frequency of writing, types of writing (transactional, expressive, or poetic). Moderating variables in quantitative studies included gender, prior achievement, and prior writing ability. Finally, the relationships between attitude towards mathematics, operationalized in a variety of ways and measured with a variety of instruments, writing to learn in mathematics, and student achievement, have been explored in many of the studies cited in this study. As noted earlier in this review, different kinds of writing activities led by different kinds of prompts help students focus on different kinds of information, different types of knowledge (Langer & Applebee, 1987). The next part of this review will examine the literature related to the different kinds of knowledge in thought to be components of mathematical knowledge, and the prompts that would focus students' attention on those different types of knowledge.

TYPES OF KNOWLEDGE

Building upon the foundations of Gagne and Briggs (1979, cited in Yildirim, Ozden, & Aksu, 2001), cognitive theorist and instructional design specialist J. R. Anderson (1983) identified three types of knowledge, declarative, procedural, and conditional, thought to make up the human cognitive system. Shortly thereafter, literacy researchers Paris, Cross, and Lipson (1984) applied this construct to the teaching and learning of reading and writing, and recognized its applicability across all knowledge domains. Ten years later, Ellis and Worthington (1994), cite the inclusion of these three forms of knowledge mentioned above as one of the top ten effective principles of teaching. In 2000, the NCTM, in the document, *Principles and Standards of School Mathematics*, cites Bransford, Brown, and Cocking

(1999), stating that, “One of the most robust findings of research is that conceptual understanding [conditional knowledge] is an important component of proficiency, along with factual knowledge and procedural facility” (p. 20).

The theoretical construct utilized for declarative knowledge was fairly consistent across studies with some variations in the way it was operationalized in specific contexts. In general, most studies built on Anderson’s (1983, 1995), defining declarative knowledge as knowledge about facts and ideas about content (Basile & Copley, 1997; Brock, 1993; Hsu, 1994; O’Ferrall, 1998; Paris, Lipson, & Wixson, 1983; Pennington & Nicolich, 1995; Sandberg, Christoph, & Emans, 2001; Yildirim et al., 2001). Some studies cited Bruner’s (1972) characterization of declarative knowledge as “knowing that” (Neuman & Roskos, 1997; Sheehan & Tessmer, 1997), while Sandberg et al. (2001) characterize this knowledge as “knowing what or which” (p. 72).

In several studies, however, the construct of declarative knowledge was extended in a variety of ways, operationalizing its use for specific research purposes. Joe and You (2001) defined this type of knowledge as it relates to metacognition after the reasoning of Schraw (2001) as the knowledge learners have about the factors that influence their own performance on tasks. Similarly, Pei and Steinbart (1994) considered declarative knowledge to be the basic understanding of the factual knowledge about internal controls subjects have over their own problem solving processes. Stanovich, West, and Harrison (1995) referred to declarative knowledge as crystallized intelligence, citing Salthouse, Kausler, and Sauls (1988), and, aligned with the ideas of Chen (1997) and Piburn (1994), specify semantic knowledge as part of declarative knowledge.

Declarative Knowledge

Several researchers extend their constructs beyond those cited above, sometimes coming into conflict with each other. Lavoie (1993) includes in declarative knowledge if-then production rules and how they are structured and applied in prediction problem-solving, as do Lorch, Lorch, and Klusewitz (1993), and Piburn (1994). In contrast, Yildirim et al. (2001) place if-then statements in the category of conditional knowledge. In addition, Piburn’s construct comes into some degree of conflict with those of Brock (1993), Yildirim et al., and Sandberg et al. (2001) on important characteristics of declarative

knowledge. While Brock, Yildirim et al., and Sandberg et al., theorize that declarative knowledge is static, unchanging in content, flexibly organized, and explicit, easily described or verbalized making it easily accessible and of which we are conscious aware, Piburn, citing Gobbo and Chi (1986) suggests this knowledge is not necessarily static or primarily factual, especially in expert knowledge bases.

In mathematics, most theorists and researchers operationalized the concept of declarative knowledge as knowledge about mathematics (Ball, 1988; Farnham-Diggory, 1994; Moenk, 2001). This definition includes factual information, such as multiplication and addition facts, and other static knowledge of facts and ideas about mathematics (Anderson, 1983; Bruner, 1972). Though declarative knowledge is often considered to be requisite to rapid and efficient mathematical computation and reasoning, it is generally considered to require only lower-level thinking and prompts for this type of knowledge will not be used in this study.

Procedural Knowledge

Of Anderson's three types of knowledge, procedural knowledge is the most coherent and uniform across the studies reviewed. The construct of procedural knowledge, again based largely on the work of Bruner and Anderson, is represented statically as knowing how (Hsu, 1994; Lavoie, 1993; Neuman & Roskos, 1997; Sheehan & Tessmer, 1997), knowledge of methods, strategies, and approaches to a skill domain (Brock, 1993; Dole, Sloan, & Woodrow, 1995; Joe & You, 2001), process skills (Basile & Copley, 1997), knowledge of how to perform a sequence of operations, or problem solution execution (Chen, 1997), and information about the various actions that must be performed in a task (Hsu, 1994). On a more dynamic level, procedural knowledge is characterized in the literature as the ability to apply a set of factual knowledge toward problem-solving (Anderson, 1983; Pei & Steinbart, 1994), encoding algorithms (Lewicki & Hill, 1994), and the execution of reading tactics, such as knowing how to skim or summarize (Lorch et al., 1993).

In mathematics, the static notion of procedural knowledge as "knowing how" is supported by the work of Farnham-Diggory (1994) and Paris, Lipson, and Wixson (1984). Liu (1995), however, situates procedural knowledge in the field of mathematics as the knowledge of mathematical symbols and knowledge of rules and algorithms for completing

mathematical tasks. This two-part, more active conception of procedural knowledge in mathematics is consistent with that of Hiebert and LeFevre (1986), who extend knowledge of mathematical symbols to include the syntactic rules for writing symbols in an acceptable form. In addition, Hiebert and LeFevre distinguish between two types of procedural knowledge: (1) that which involves the transformation of an equation in one form to an equation in an answer form by following a series of symbolic manipulation rules, such as solving an algebraic equation (i.e., $2x+8=12$; $2x=4$; $x=2$); and (2) that which is a type of problem-solving strategy or action that involves the manipulation of concrete objects, visual diagrams, and other entities. The first type is found most often in school mathematics, whereas the second type, though sometimes found in school tasks like geometric constructions with straightedges and compasses, is most often used by preschool children and by others on non-school tasks. Finally, Hiebert and LeFevre emphasize that it is “the sequential nature of procedures that probably sets them most apart from other forms of knowledge” (p. 6). One of the most distinguishing and important features of procedural knowledge is its structural nature, where subprocedures are embedded in hierarchical arrangements that comprise superprocedures. The relationship between subprocedures is primarily linear, whereas there are many kinds of relationship found in conditional and other types of knowledge.

Conditional Knowledge

Of the three types of knowledge originally theorized by Anderson (1983), constructs of conditional knowledge are the most widely varied. Though theorists and researchers disagree on the exact nature of this type of knowledge, they consistently agree that it does involve not only knowledge of content, but also the many and varied relationships between the different types of knowledge. In addition, conditional knowledge is often paired with understanding and meaning, rather than skills and rote learning. For this reason, conditional knowledge is often referred to as conceptual knowledge.

Conditional knowledge, as represented by Paris, Lipson, and Wixson (1983) after Anderson (1983), is knowing why a strategy works or when to use one skill or strategy as opposed to another (Dole et al., 1995; Hsu, 1994; Joe & You, 2001; Lorch et al., 1993). Yildirim et al. (2001) fundamentally agree with Anderson’s original construct, representing

conditional knowledge as that which includes relational rules, such as if-then statements and networks of condition-action sequences, while Sheehan and Tessmer (1997) echo this idea by including causal principles or functions in this category of knowledge, after Jonassen, Tessmer, and Hannum (1999). Liu (1995) utilizes the construct of conceptual knowledge, a label also used by Anderson (1983). In this construct, conceptual knowledge is envisioned as a connected web of knowledge linking discrete pieces of information to a larger system. Hiebert and LeFevre (1986), characterize this type of knowledge as “rich in relationships” (p. 3). Moenk (2001) refers to conceptual knowledge as well, following the theories of Farnham-Diggory (1994), who identified five separate types of knowledge: declarative, procedural, conceptual, analogical, and logical. In Moenk’s study, conceptual knowledge was that of, “why things work and how they fit together” (p. 23), a definition that closely related to those mentioned above. Finkel (1996) refers to a type of knowledge beyond declarative and procedural that may be analogous to that of conditional knowledge, but labels it “principled knowledge”, knowledge that is understood in such a way that it can be used in a variety of contexts, or transferred to various situations.

Perhaps the best way to understand conditional, or conceptual, knowledge is by comparing it to declarative and procedural knowledge. Hiebert and LeFevre (1986) liken conceptual knowledge to meaningful learning, “generated as relationships between units of knowledge [declarative and procedural] are recognized and created” (p. 8). Whereas declarative and procedural knowledge may or may not be learned by rote and consist of facts stored in memory as isolated bits of information, conceptual knowledge cannot be learned by rote. It must be learned meaningfully and the learner must recognize its relationship to other pieces of information. Conceptual knowledge is always linked to understanding while procedural and declarative knowledge may be applied without understanding. Finally, declarative and procedural knowledge are necessary but not sufficient components of conceptual knowledge.

The importance of developing conditional/conceptual knowledge has become an international topic of discussion in the mathematics education community. In their study of the Third International Mathematics and Science Study, Stigler and Hiebert (2004) found that one of the major differences between mathematics instruction in the United States and in countries that scored higher than the U.S. was that the higher-scoring countries spent

substantially more time on *making connections* problems (conditional knowledge, including conceptual knowledge and metacognition), while teachers in the United States spent more time on *using procedures*, or practicing procedures (declarative knowledge). In addition, even when American teachers began with *making connections* problems, they transformed them into procedural problems without the connections. In their conclusions, the researchers found that 8th grade American mathematics students spent most of their time practicing procedures, rarely spending time developing mathematical concepts.

While often mixed, results of studies of writing in mathematics have generally shown that writing has a positive effect on the acquisition of procedural and conceptual knowledge in mathematics. Jurdak and Zein (1999) found that journal writing in mathematics had a significant effect on conceptual and procedural understanding, while others found that writing to learn activities may have been more beneficial to concept development than to computation and application, aspects of procedural and conditional knowledge (Davidson & Pierce, 1988; Mower, 1996; Resnik, 1992; Stewart, 1992). Lim and Pugalee (2005), whose prompts and assessments in a grade 10 applied math class focused on procedural knowledge, asking students to describe their problem-solving processes step-by-step, found that writing helped students develop conceptual understanding of mathematical concepts and processes, emphasizing the interrelated nature conceptual and procedural knowledge.

Metacognitive Knowledge

Anderson's (1983) three forms of knowledge, declarative, procedural, and conditional (or conceptual) are cognitive processes and part of a unitary theory of cognition held by many (Marzano, 1985). Other theorists believe that there is no uniform principle of growth and learning, no general purpose learning strategy, including Chomsky (1980) and Gardner (1983), and still others have found evidence that there may be domain-specific thinking skills in which specific disciplinary knowledge is applied to content-specific thinking tasks, as well as general thinking skills that apply to everyday problem situations (Smith, 2002). In a unitary theory, higher processes, such as memory, language, imagery, deduction, and induction are all part of an underlying system of cognition and explain constructs such as problem solving, inference, and general schema systems (Marzano, 1985). Marzano extends Anderson's theories by positing two cycles of the thinking process in which information is

recognized and acted upon based on the conditions of the task. The first cycle includes declarative, procedural, and conditional knowledge. This cycle, according to Marzano, exists within a larger cycle that includes attention focusing, goal setting, epistemic thinking (thinking that situates a problem within a person's world view), and task monitoring. Marzano's larger cycle, which he calls learning to learn, has many aspects in common with the more contemporary notion of metacognition.

While many definitions of metacognition exist in the literature, "all emphasize the role of executive processes in the overseeing of and regulation of cognitive processes" (Livingston, 1997, p. 1). Executive processes are those that are involved in the self-regulation of thinking and learning. Most often associated with the work of Flavell (1979), metacognition "involves an awareness of the mental processes and strategies [personal resources] required for the performance of any cognitive endeavor" (as cited by Schmitt & Newby, 1986, p. 29). According to Schmitt and Newby, the learner needs the three types of knowledge described above for metacognitive awareness and gives the following example to illustrate the relationships between declarative, procedural, conditional, and metacognitive knowledge:

Suppose that a proficient learner is faced with the task of reading an article about rodents, about which he must prepare a simple oral report. The learner demonstrates declarative knowledge of personal resources when he thinks, "I already know something about rodents," and "I usually remember informational-type text easier than I do stories." Declarative knowledge of task characteristics is evident when the learner thinks, "Reporting on the information in this article will require that I understand and remember it," and "this type of text usually consists of ideas and supporting details." In order to match an appropriate strategy with the task, the learner class on his store of task-related declarative and conditional knowledge, thinking, "I know that outlining and summarizing informational text is a good strategy for organizing and remembering the information because it forces me to identify the important details, so it should work well in this case," (encompassing the what, when, and why). Procedural knowledge is what accounts for the learner's ability to execute the skill of summarizing or outlining. (p. 30)

In this example, the processes of planning, monitoring, and revising comprise the metacognitive, or regulatory component. The cognitive processes that function to control information processing and task performance from the outset are involved in the planning aspect of metacognition. Planning is goal oriented and involves the initial selection of strategies for solving the problem at hand, such as the choice to use summarizing and outlining in the example above. Monitoring involves the ongoing regulatory control of those

process and is involved in checking and evaluating whether selected strategies are working, whether the task is being performed adequately, and whether the student is understanding the information involved in the task. Revision occurs in response to the monitoring process when the learner finds the chosen strategies are not working, the problem is not being addressed adequately, or that he is not understanding the information. The more competent a learner becomes at metacognitive strategies, the more these processes occur below the level of consciousness and the more efficient and precise the performance of related tasks become (Gagne, 1983; LaBerge & Samuels, 1974; Schrifflin & Schneider, 1977).

In mathematics, Schoenfeld (1987) describes three ways metacognition is related to learning, especially the construction of understanding. First, metacognitive knowledge allows students to think about how their own beliefs about mathematics shape the way they do mathematics, making the learning of mathematics more connected to students' lives and experiences, not just static, formulaic information unconnected to the real world. This kind of reflection increases the chances for students to construct their own understanding. The beneficial role of writing in the construction of meaning is widely accepted (Applebee, 1981; Britton et al., 1975; Emig, 1977; Flower, 1989; Mayher, Lester, & Pradl, 1983; Miller & England, 1989). Several studies of writing in mathematics have examined the role students' beliefs about mathematics in learning and its fostering of a constructivist environment in the classroom. Many found that when students reflected on their own beliefs about mathematics and the learning of mathematics, the learning environment became more open to discussion and active, resulting in more positive attitudes towards learning according to teacher and students self-reports (Millican, 1994; Scott et al., 1992; Shield & Galbraith, 1998).

Second, Schoenfeld (1987) theorized that a person's approach to a task and his understanding of how to solve that task are affected by the extent to which they can realistically assess their own ability to solve the task. In other words, the way you solve a problem depends on how much you know and on having a good sense of what you know and don't know. He found that, while young children had unrealistic ideas of how much they could memorize, their assessment of their estimates of their abilities to memorize grew more accurate as they matured. The findings of research of writing in mathematics also support this view (Brown, 1995; Kasperek, 1993; Lim & Pugalee, 2005).

The third way in which metacognitive knowledge may improve student understanding according to Schoenfeld (1987) is through self-awareness and self-regulation. This process includes keeping track of one's own progress when working on a task, including checking to see whether the answer is reasonable and/or correct at any step in the process. Schoenfeld adopts a management approach when describing this kind of metacognitive knowledge. He states:

Aspects of management include (a) making sure that you understand what a problem is all about before you hastily attempt a solution; (b) planning; (c) monitoring, or keeping track of how well things are going during a solution; and (d) allocating resources, or deciding what to do, and for how long, as you work in the problem. (pp. 190-191)

Research in the area of writing supports the view that these aspects of self-awareness and self-regulation are evident in the thinking processes of the most successful problem-solvers (Lester, 1989; Simon, 1987), a finding also supported in studies of writing and problem-solving in mathematics (Artz & Armour-Thomas, 1992; Deseote, Roeyers, & Buysse, 2001; Lim & Pugalee, 2005; Linn, 1987; Quinto & Waever, 1983). In addition, studies have found that instruction in metacognitive strategies that included monitoring and evaluation had a positive effect on student achievement (Maqsud, 1997; Zan, 2000) and problem solving (Kramarski, Mevarech, & Arami, 2002).

Mathematical Reasoning

A discussion about types of knowledge in mathematics is incomplete without discussing the role of mathematical reasoning, or knowledge of the domain-specific relationships between declarative, procedural, and conditional/conceptual knowledge in mathematics. The NCTM (2000) describes mathematical reasoning as "...developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas" (p. 56). People who reason mathematically recognize and attend to patterns, consider prior and new knowledge in terms of mathematical structures or regularities in both real-world situations and symbolic objects. Important aspects of mathematical reasoning include looking for reasons patterns occur (asking why) and engaging in conjecture and proof, either formal or informal. According to Loewenberg Ball and Bass (2003), "the notion of mathematical understanding is meaningless without a serious emphasis on reasoning" (p. 28) and is related to mathematical understanding as comprehension is to reading. As stated

above, declarative knowledge, including knowledge of procedures, is useless without making the connections between when and why that knowledge is most efficiently and effectively used. Loewenberg Ball and Bass make the case that “reasoning” is precisely that: “making mathematics known in useful and useable ways” (p. 29).

Mathematical reasoning is a basic mathematical skill that describes the relationships between declarative, procedural, and conditional knowledge without which knowledge of mathematical ideas and procedures cannot lead to flexible and diverse problem-solving. Without mathematical reasoning, students whose declarative knowledge about procedures has faded will not be able to make the connections necessary to rebuild that knowledge in an accurate and useful way. Many students who have not developed mathematical reasoning and rely on their memories of rules and procedures to solve problems make senseless errors. For instance, many people know how to move the decimal point when multiplying decimals by counting the number of numbers to the right of the decimal in the problem and move it the same number to the left in the answer. However, many do not know why or what it means and will apply the same rule when they forget what procedures to apply when adding or subtracting decimals. Those who learn mathematics through reasoning, then, are more able to use mathematics in diverse situations and use declarative, procedural, and conceptual knowledge to build new mathematically sound procedures and rebuild those that have been forgotten. It must be remembered, however, that the base of mathematical reasoning is knowledge of facts, concepts, and procedures, as well as the meaning of mathematical terms and expressions, declarative knowledge that serve as resources available for active use in problem solving and other mathematical acts.

Loewenberg Ball and Bass (2003) describe two foundations of mathematical reasoning, a shared body of knowledge within the classroom, or community, and mathematical language. The first foundation, shared knowledge (Yackel & Cobb, 1996), refers to the “meanings, norms, and ideas that are negotiated and used as common within a classroom” (Loewenberg Ball, & Bass, p. 31). For example, when students in a class work on a common problem and alternative strategies are made public through discussion, that knowledge, those strategies, become a part of the shared knowledge of the classroom. From a broader perspective, professional mathematicians draw on the public knowledge of the discipline, such as axioms when proving geometric theorems or negotiated structures when

representing functions (the horizontal axis of a coordinate system relates to the domain and the vertical axis relates to the range). Shared knowledge is both part of the students prior knowledge, and, as it is developed within the community of learners, becomes prior knowledge as students engage in public mathematical reasoning

The second foundation of mathematical reasoning, mathematical language, is an essential component and product of shared knowledge. Domain-specific terms and syntactic and semantic structures, including the nature of symbolic notation and the rules for symbolic manipulation are the resources from which students may draw in order to reason mathematically. Every discipline has its lexicon of domain-specific terminology and, while it is important to learn the meanings of mathematical vocabulary, it is the semantic structures of the discipline that convey meaning upon those terms, including its ways of arguing, making conjectures, and drawing conclusions. Lemke (1990) describes language as a system of resources for making meanings and emphasizes the importance, once again, of the connections we make within and between our cognitive resources: “We need semantics because any particular concept or idea makes sense only in terms of the relationships it has to other concepts and ideas” (p. ix). Connolly and Vilardi (1989) also emphasizes the importance of learning mathematical language, especially through writing to learn activities, in the development of conceptual understanding. He cites Rorty (1982):

If there is one thing we have learned about concepts in recent decades, it is that to have a concept is to be able to use a word, that to have a mastery of concepts is to be able to use a language, and that languages are created rather than discovered. (p. 222, as cited in Connolly & Vilardi, 1989, p. 5)

In summary, one view of cognitive theory (Anderson, 1983) posits three types of knowledge as components of two cycles of thought. The inner cycle is composed of declarative, procedural, and conditional (often referred to as conceptual in the literature), each intricately related and somewhat overlapping in their relationships. A larger, governing cycle of the thought process consists of self-regulating, self-monitoring processes and is most often referred to in the literature as metacognition, or metacognitive knowledge. In mathematics, conceptual understanding is thought to be founded on mathematical reasoning and language which define domain-specific relationships between the three types of knowledge described by Anderson. The next section describes the kinds of writing in which students might engage in order to develop mathematical knowledge, especially procedural,

conditional, and metacognitive knowledge, and the prompts that develop these types of writing.

STUDENT WRITING AND WRITING PROMPTS/WRITING TASKS

As noted earlier in this review, Langer and Applebee (1987) found that the choice of writing prompt played a role in determining the types of knowledge students thought about and took away from their experiences. Much of the literature on writing prompts focuses on the topics involved in writing assessment, especially since the advent of large-scale writing assessments (Oliver, 1995). Researchers agree that poorly written prompts do affect student writing negatively (Murphy & Ruth, 1993) and that the form and type of prompt does affect the content and quality of student writing (Hoetker, 1982; Oliver, 1995). It is also clear from the literature that effective writing to learn activities are not arbitrary assigned, but designed by the teacher to meet particular learning objectives (Langer & Applebee, 1987; Penrose, 1989; Reese & Zielonka, 1989). Teachers must “give assignments with predetermined pedagogical purposes that meet short or long term learning objectives for the class” (DeNight, 1992, p. 7). In mathematics, both short-term and long-term instructional goals include the development of declarative, procedural, conceptual, and metacognitive knowledge through mathematical reasoning.

While the literature surrounding prompts in mathematics is mainly comprised of general descriptions of the prompts used (i.e., creative, expressive, poetic, narrative, informational, and transactional) and the benefits of writing in general, a few studies have focused on the types of writing students do in mathematics, and types of prompts and their effects on student writing and understanding. Miller and England (1989) chose specific types of prompts in order to examine what the teacher could learn from student responses to carefully designed prompts. In their exploratory study of high school students in Louisiana, several findings were of interest, but few were specifically related to the type of prompt used. Prompt-related findings included error patterns and content-specific misconceptions that led to adjustments in subsequent instruction. For instance, a discussion of one prompt led teachers to conclude that the choice of language was unfamiliar or vague and created confusion and misconceptions in students’ understanding of the material. No comparisons were made between the types of prompts and the nature of students’ responses.

One study of particular interest was that of Clarke et al. (1993). In their four-year longitudinal study of expository journal writing in grades 7-11 in Great Britain, they found that students wrote in three basic modes, Recount, Summary, and Dialogue. In Recount writing, students wrote more about the activities of the class than the mathematics, such as “We did a warm-up, then worked on a problem, then wrote in our journals.” In Recount writing, no mathematical knowledge was evident until they began to reach a transition phase between Recount and Summary. Students who engaged in Summary writing began with relating the main points of the lesson and progressed until those points included not only declarative and procedural knowledge, but also conditional knowledge, relating both when and why mathematical terms and procedures were used and comparing different techniques and situations. The most important feature of Dialogue writing was students’ use of internal dialogue, where they employed metacognitive knowledge in order to analyze and synthesize the lesson. In the examples given by the authors, students who engaged in Dialogue writing asked themselves questions and self-corrected their thinking in writing, producing evidence of self-monitoring and correction. While the results of this study provide insight into the types of writing students might engage in mathematics and even a progression from lower-order to higher-order thinking, no information was provided about the prompts that were used or how instruction might have encouraged the progression of students’ mathematical writing.

A later study also shed some light on the types of mathematical writing students produce as a result of writing to learn activities (Shield & Galbraith, 1998). In their study of expository writing of students from grades 3-8 in Australia, participants were asked to either describe or explain their thinking as it related to mathematical task. The authors used the features of Leinhardt’s (1987) model of an “expert” explanation to examine and describe student writing. These features include identifying the goal, using examples of the given case, using multiple representations of the problem and relating them together in some way, identifying conditions of use and non-use, justifying their thinking through known principles, cross-checks of representation, or other means of logical argument, and linking new concepts to old. The process of linking new knowledge to old and between representations is known in the literature as elaboration (Swing & Peterson, 1988) and has been shown in a variety of contexts to be related to improved comprehension and retention of new material (Anderson,

1995) by allowing learners to integrate new material into existing knowledge structures. Through writing tasks that encourage elaboration, new material becomes more meaningful and conceptual understanding is likely to increase (Bradley, 1990). In addition, elaborative processing is likely to lead to increase in the ability of students to apply new knowledge in diverse situations (Hamilton, 1990; Mayer, 1980). Shield and Galbraith found that students in general wrote explanations that focused on procedures without elaboration, without connections, and that this type of writing closely paralleled their experiences with the writing found in their textbooks. Even after attempts were made to model and encourage elaborative process in writing, students' writing remained the same. The researchers concluded that students held a certain belief about mathematical writing that was persistent and consistent with their previous experiences in mathematics education. Recalling Schoenfeld's (1987) first category of metacognitive knowledge in which students beliefs shape the way they do mathematics and the need for students make connections between all types of knowledge, the findings of this study support the need for instruction in both conceptual and metacognitive knowledge.

For the purposes of the present study, then, three prompts will be used--procedural, summary, and metacognitive. In order to provide the most effective instruction for all students, non-writing tasks will be designed to include all types of mathematical knowledge, declarative, procedural, conditional/conceptual, metacognitive, and mathematical reasoning. Since the nature of the writing prompt determines the nature of the writing task, the two terms will be used interchangeably throughout this study. Procedural writing tasks will ask students to focus on procedural knowledge and require not only explanations of procedures, or algorithms, but also the use of elaborative processes, making connections between multiple representations and/or using mathematical reasoning by justifying steps (Leinhardt, 1987; Loewenberg Ball & Bass, 2003). Since metacognition encompasses a wide variety of cognitive management skills and processes, metacognitive prompts in the present study will focus students' attention on one of the processes found most consistently in the literature, self-monitoring. These tasks will encourage students to engage in self-checking behaviors by asking themselves questions such as "Does my answer make sense?" and "How can I tell if my answer is correct?"

While the topics of procedural and metacognitive knowledge have been addressed previously in this review, a short review is in order here. The literature surrounding writing to learn supports the use of summary writing as a means to improving retention and to engage students in the processes of analysis as they “evaluate information in a text and make decisions about what is most important” (Fearn & Farnan, 2001, p. 398). In summarizing text, including classroom discussions and activities, students are also required to synthesize by connection new information withhold and rearranging the information in a way that makes sense to them. In addition, summarizing is thought to help students clarify meaning and recognize the significance of new information (Brown, Campione, & Day, 1981; Hidi & Anderson, 1986). Johnson (1983) suggests six components in the development of a summary, including understanding individual propositions and other pieces of information, making connections between those pieces of information, recognizing the structure of the original text, remembering the information, selecting the important information, and reconstituting the information into a verbal representation that is both concise and coherent.

In the literature from writing to learn in mathematics, summary writing was found to provide the same kinds of benefits as those mentioned above and those mentioned in the writing to learn in mathematics section of this review. Brown (1995) found that weekly summary writing helped student clarify meaning, organize their thoughts, and synthesize the mathematics content of the week. Student summaries also helped the instructor understand student thinking and attitudes. Azzolino (1990, as cited in Brown, 1995) found that summary writing showed students and the teacher evidence of misunderstandings, guiding instruction. Clarke et al. (1993) concluded that, while summary writing gave evidence of students’ abilities to gain an overall picture of the mathematical situation, delineate important information, and focus on key steps in their work, this type of writing did not show the level of self-reflection as those who engaged in more metacognitive processes.

Summary prompts in the present study, then, will focus students’ attention on the main points of the text of the class, the topics under discussion, which may include procedures, vocabulary, multiple representations, conceptual understanding, and/or specific classes of problems and the most common and efficient procedures related to solving them. Again, in order to encourage more than just the description and rote memorization of mathematical facts and processes, students will be asked to engage in higher level thinking

by making connections between all types of knowledge, giving evidence that they know when and why to use specific procedures or problem-solving processes, and conceptual understanding.

SELF-CONCEPT OF ABILITY

Over the years it has been widely theorized that attitude has a positive effect on achievement in school (Fennema & Sherman, 1976; Schofield & Start, 1978; Sudyam & Weaver, 1975). However, like writing to learn in mathematics and student achievement, the relationship between the two variables is not clearly defined (Schofield, 2001) and the evidence for a positive relationship between the two is not consistently supported by research. Early reviews of the literature in mathematics have found that effects, while statistically significant, are not large (Aiken, 1970), or that the correlation between attitudes in mathematics and achievement account for only a small percentage of the common variance between these variables (Robinson, 1975). More recent studies have resulted in similar findings (Quinn & Jadav, 2001). Studies involving crossed-lagged panel correlation, a quasi-experimental method used to reveal causal predominance between two variables, such as achievement and attitude, measured at two points in time (Minato & Kamada, 1996) have also been inconsistent or shown no significant causal predominance between attitude and achievement.

The reason evidence supporting the existence of a positive relationship between attitude and achievement is inconsistent may be the complexity of the constructs involved, especially the existence of a wide variety of moderating variables and in defining both attitude and achievement. Many studies examining the relationship between attitude and achievement have theorized or identified moderating variables between these two major variables, reporting that attitude may have an indirect positive effect rather than a direct effect on achievement (Hammouri, 2004). In their research aimed at determining the causal relationship between attitude toward mathematics and achievement in mathematics, Ma and Xu (2004) confirmed previous research that found prior student achievement in mathematics, “demonstrated causal predominance over attitude across the entire secondary school” (p. 256), with high achievement (elite status) as a moderating variable. Kulubya and Glencross (1997) found several moderating variables including language of instruction,

inadequate instructional aids, poor social and academic backgrounds, grade level, and whether students' math class was required or optional. In addition to prior mathematics achievement, studies have identified other significant moderating variables including gender (Nuyangeni, & Glencross, 1997; Schofield, 2001), parent beliefs about the value of mathematics and expectations (Boehnke, 2005; Brookover et al., 1964), peer perception of ability (Brookover et al.), math perception (Tsao, 2004), mathematics anxiety (Boehnke, 2005; Eccles Parsons, Adler, & Kaczala, 1982), student/teacher relations (Midgley, Feldlaufer, & Eccles, 1989), grade level, especially between elementary school, transition periods such as elementary school to middle school and middle school to high school, high school, and post-secondary school (Ma & Xu, 2004; Midgley et al., 1989; Schofield, 2001), and cognitive factors including learning styles, visual and spatial ability, use of cognitive and metacognitive strategies (Higbee & Thomas, 1999).

In addition to moderating variables, one of the main difficulties found throughout the literature that attempts to determine the effects of attitude toward mathematics on student achievement in mathematics is that there are no generally accepted definitions of achievement or attitude or standardized measurement instruments for either construct, though a few occur more or less frequently in the literature. For instance, achievement may be defined in many ways. In the literature, achievement has been operationalized as developing problem-solving abilities, knowledge and/or use of correct procedures, knowledge and/or use of conceptual knowledge, or computational accuracy. Measures of achievement vary in the literature from scores on standardized, norm-referenced tests, scores on standardized tests developed by publishers of textbooks or state/national achievement tests, teacher-created materials, or teacher observation.

While defining and measuring achievement can be problematic, attitude is even more difficult to define. It is a concept that may be comprised of many subconstructs, including perception of the teacher, enjoyment in mathematics, motivation, epistemological beliefs about mathematics, perception of mathematics materials, and many other motivational, perceptual, and personality traits. In addition, the researcher may choose to sum the scores of an instrument and call the variable simply "attitude toward mathematics," while others focus on the effects of one or more of the individual subconstructs on achievement, making it difficult to draw any conclusions about the current state of the evidence with regard to the

effect of attitude on student learning in mathematics. In an interesting study that draws many of these complexities together, McCoy (2005) found attitude, measured by the sum of the scores on an instrument that includes subtests of confidence in using mathematics, perceived usefulness of mathematics, and math anxiety in its construct of attitude toward mathematics, had no significant effects on achievement as measured by state achievement tests, though attitude did have a significant effect on student grades in algebra. In fact, post-attitude scores were significantly lower at the end of the study for each of the subtests included in the attitude measurement instrument. This decline in attitudes towards mathematics, specifically students' self-concept of ability in mathematics, during the transitional grades of middle school was also identified by Eccles et al. (1989) as a possible predictor of achievement in mathematics. Thus, self-concept of ability is a phenomenon deserving of further study in order to find effective interventions to prevent such a decline.

As early as 1964, researchers conceptualized self-concept of ability as a student's "conception of his own ability to learn the accepted types of academic behavior; performance in terms of school achievement is the relevant behavior influenced" (Brookover et al., p. 271). Later, Bandura (1986) examined the relationship between global self-conceptions and perceptions of self-efficacy in particular situations, noting that composite self-images are not necessarily good predictors of how people may behave in a situation in which they believe they are not competent. Students who have high global self-images may still perceive themselves to be less capable in specific content areas, such as mathematics, science, or history. In an early study of seventh graders in an urban school system, Brookover et al. (1964) examined the relationships between global self-concept of ability, specific self-concept of ability, and achievement in several academic domains and found a significant and positive correlation between self-concept of ability in specific subject areas and achievement in those areas. In addition, they found that specific self-concept of ability and achievement correlations were higher than those for general, or global, self-concept of ability for males in mathematics, social studies, and science, but only in social studies for females. Correlations between specific self-concept of ability and achievement in English for both males and females were lower, though not significantly. More recent studies have focused attention on determining the causal ordering of self-concept of ability and the developmental aspects of self-concept of ability, especially in transitional periods.

Three possible relationships between self-concept of ability and achievement have been identified and modeled in the literature. In the self-enhancement model, the level of self-concept of ability determines the level of subsequent achievement. Social cognitive theory points out that people are more likely to choose to persevere and expend effort in areas in which they perceive themselves to be the most able (Schiefele & Csikszentmihaly, 1995). It follows logically that students are more likely to make achievement gains in subjects in which they choose to expend the effort necessary to address and complete assigned tasks. In other words, students who achieve more in a given subject perceive themselves as more efficacious, leading to even higher achievement in that subject through increased effort and perseverance. In the skills development model, the level of success, or achievement, determines students' self-concepts of ability. In other words, "self-concept is primarily the result of past achievement rather than a cause for subsequent achievement" (Helmke & vanAken, 1995, p. 624). Finally, in the reciprocal model, self-concept of ability and achievement affect each other and one does not necessarily precede the other.

As in the cases of writing and achievement and attitude and achievement, the results of studies in this area have been mixed. There are studies in mathematics that support the skill development model (Newman, 1984, as cited in Helmke & vanAken, 1995), the self-enhancement model (Shavelson & Bolus, 1982, as cited in Helmke & vanAken, 1995), and the reciprocal model (Skaalvik & Hagtvet, 1990). In their study of German elementary school students using structural equation modeling (SEM), Helmke and vanAken (1995) found support for the skills development model in mathematics, especially when using both tests and grades as indicators of achievement. At least in elementary school, it seems that self-concept of ability "does not significantly contribute to the prediction of subsequent achievement" (Helmke & vanAken, 1995, p. 624). In a study of the transition between elementary and secondary school mathematics students using a traditional autoregressive model, however, found "significant paths representing both directions of influence between self-perceptions of academic ability [self-concept of ability] and achievement" (Silverthorn, DuBois, & Crombie, 2005, p. 212). Overall, the paths from achievement to self-perception of ability disappeared, however, when an alternative model was used while paths from self-perception to achievement remained, though smaller in magnitude. The researchers attributed these findings to the effects of stable sources of variance in the constructs of

self-perception of ability and achievement more evident in the second model, such as parental attitudes and beliefs, cognitive ability, and demographic factors. In mathematics, however, self-perception of ability influenced achievement, but not vice versa, possibly due to the very structured, sequential nature of the discipline. In science, the content of which tends to vary from year to year, self-concept of ability appears to be less likely to influence achievement. The researchers concluded that the effects of self-perception of ability and achievement on one another may change at different times during the schooling process. In other words, as a student's awareness of her self-concept of ability changes over time, the potential for that perception to serve as a resource to effect achievement also changes, a finding supported by the work of Eccles et al. (1989). At the same time, the stable components of achievement, such as academic skills and ability or course grades, may have a greater effect on self-concept of ability than less stable factors, such as immediate task performance.

SUMMARY

While the arguments for writing to learn in mathematics seem compelling from a theoretical point of view, the evidence supporting its efficacy in promoting higher achievement, especially in mathematics, is mixed at best. Qualitative data yield especially positive results through researcher observation and participant self-reports and many benefits, both cognitive and affective, have been reported. Quantitative data have been less consistent, possibly due to differences in definitions in terms and in methodologies, especially in terms of measurement and treatments.

Writing to learn may be used to focus students' attention on the types of knowledge teachers want to address, types that may be specific to particular disciplines or areas of study within a given discipline, such as mathematical reasoning. Writing prompts and tasks may be crafted to tap into and affect the development of and relationships between several different types of knowledge identified in the literature as declarative, procedural, conditional, and metacognitive.

In addition to cognitive development, the literature supports the idea that attitude affects achievement, though the relationship between these two constructs is complex. The results of studies in this area have, again, been mixed and often inconclusive for the same

reasons given for the results in writing to learn. One of the reasons evident from the literature is the complexity of the construct of attitude. Attitude in mathematics may be considered from either a global perspective or from the perspective of its various subconstructs, such as beliefs about the nature of mathematics or the value of mathematics in an individual's life. One subconstruct of attitude towards mathematics that has shown some promise for efficacy in affecting student achievement is self-concept of ability. Again, the results of studies examining the effect of self-concept of ability on achievement have been mixed, though three distinct relationships have emerged: self-concept primarily affects achievement, prior achievement primarily affects self-concept, or that a reciprocal relationship exists between the two variables. In addition, the literature shows that the relationship between the two variables may change over time, depending on students' developmental needs, especially during transition periods in schooling.

The purpose of this study is to examine the effects of writing to learn prompts that target specific types of knowledge in mathematics on student achievement both quantitatively and qualitatively, as well as to explore the relationship between writing and self-concept of ability in low-achieving mathematics students.

CHAPTER 3

METHODOLOGY

As described in previous chapters, the results of the effects of writing to learn tasks in mathematics on student achievement have been mixed. Several quantitative studies report no significant difference in achievement on teacher-created assessments between students who write to learn in mathematics and those who do not (Baker, 1994; Giovinazzo, 1996; Jurdak & Zein, 1999; Porter & Masingila, 2000), while a few found some significant differences between groups on some measures, such as one or two chapter tests, but not on others (Kasperek, 1993; Resnik, 1992). Studies that examined student achievement on standardized tests also found no significant differences between those who write in mathematics and those who do not (Brown, 1995; Henn, 1989). Qualitative results tend to be more positive, with students and teachers reporting increases in understanding of concepts and skills (Kasperek, 1996; Mower, 1996), though no achievement data were used to support these findings.

While most studies describe the types of tasks employed and have identified various categories of student responses to writing tasks, such as simple recount, summary, description, procedural, and metacognitive, none have specifically compared the effect of different types of tasks on student learning. This study seeks to compare three types of short writing tasks, description, procedural, and metacognitive, on student achievement on teacher-created assessments.

In this chapter, the characteristics of the participants involved in this study, the relationship of those participants to the general population, and the sampling procedure employed are discussed first. Next, data collection instruments are described, including information related to their validity and reliability, followed by data collection timetables and procedures. Additional procedures related to the study, including decision rules for inclusion or exclusion of data or participants are also specified. Following the description of data collection procedures, topics related to the analysis of the data are described, including a restatement of the research questions and analytical procedures to be followed. Finally, the limitations and delimitations of the study are presented.

PARTICIPANTS AND SAMPLING PROCEDURES

The sample for this study was comprised of 81 eighth grade students attending a suburban middle school in a large, mainly urban school district in Southern California. All participants were enrolled in Middle School Algebra, a class designed for low-achieving 8th grade students who had been identified as needing an extra year of curriculum support prior to taking algebra in 9th grade.

All students in the study were identified as performing “below proficient” levels in mathematics according to the California Standards Tests (CSTs). Approximately 17% of the students in the study were performing at the Basic level according to California Standards Tests (CSTs). Forty-seven percent were performing Below Basic, and 36% Far Below Basic according to CST scores. Most were also identified by the previous year’s teachers as needing additional support in mathematics, and many had scored below proficient levels in language arts on the CSTs. Approximately 53% of the students received free lunches and 15% received reduced-price lunches. In addition, 70% of the sample was Latino/Hispanic, 12% of the students were African American, 16% were Caucasian, and 1% (1 student) were Asian. Finally, approximately 44% of the participants were male.

Three sections of Middle School Algebra for the 2006-2007 school year were involved in this study. All classes were at the same school site and taught by the same teacher as part of her ordinary teaching assignment. The teacher had taught the same kind of class using the same curriculum for the two previous school years. Each class had a typical enrollment of 25-28 students. Since writing to learn literature has shown to have some positive effect in many qualitative and quantitative studies, all classes involved received each treatment during the study for ethical reasons. The procedures for data collection and research design are further described in the next section of this paper.

The sample for this study was one of convenience and did not reflect the typical socioeconomic status of the school or the achievement level of the majority of students (64%) at the school who perform at or above proficient on the CSTs. These students, however, did represent the 39% of 7th graders who fall below state standards in the district, the 44% of California students falling below the level of Proficient on state standards in 7th grade, and the 56% of economically disadvantaged students falling below the level of Proficient on state standards in 7th grade who enter 8th grade at a significant disadvantage in

mathematics achievement. In addition, this sample represented nearly 100% of the site's population of 8th grade students falling below proficient in mathematics. As a result, while the findings of this study are not generalizable to the general population of students at the school, district, or state levels, it is likely to provide useful information about the efficacy of various types of writing to learn tasks on the large number of low-achieving students in our schools.

DATA COLLECTION INSTRUMENTS

There were several instruments used for data collection in this study, including student records for demographic data, a Four Pile assessment of writing ability, the Minnesota Mathematics Attitude Inventory (MAI; 1972) for assessing student attitudes towards mathematics before and after treatments, teacher-created pre-and post-tests of student content knowledge, student self-reports of the efficacy of writing treatments, student comments in class as reported by the teacher, and student interviews.

The Minnesota Mathematics Attitude Inventory was administered as a pre-and-post assessment of student attitudes towards mathematics. The MAI was designed to measure secondary students' attitudes towards mathematics and consists of 48 questions that address the following eight constructs: perception of the mathematics teacher, anxiety towards mathematics, value of mathematics in society, self-concept of mathematics, enjoyment of mathematics, motivation in mathematics, perception of the mathematics class regarding the learning process, and perception of mathematics materials. Since the construct *perception of the mathematics teacher* cannot be measured in terms of change (the students did know the characteristics of the teacher at the beginning of the school year), the instrument was shortened to a total of 42 questions. Students responded to a 4-part Likert-type scale (ranging from Strongly Agree to Strongly Disagree). Both positive and negative items were included in each scale. The entire MAI (minus the perception of the mathematics teacher scale items on the pre-test) were administered prior to and at the end of the study. However, due to time constraints, only the self-concept of ability subscale was used in final data analyses. During the study, only the items that related to the self-concept of mathematics scale were administered following the first two units of study, providing ongoing evaluation of students' self-concepts of ability prior to and following each treatment period.

The validity of the MAI was documented in both California and Indiana through stratified random sampling on the basis of community size, “where the number of schools selected from communities of a certain size was proportional to the total state population living in communities of that size” (Sandman, 1979). A total of 5034 students in grades 8 through 12 from 413 schools contributed to the validation of the instrument during the two years 1972 and 1976. The Cronbach’s alpha calculated for the self-concept of ability for this study was .84, and indicated that the scale was reliable for this sample. A copy of the instrument is in Appendix A.

Assessment of prior writing ability will be conducted at the beginning of the study, prior to treatment administration, using a form of the Four Pile Method developed by two well-respected researchers in the field of writing research, Drs. L. Fearn and N. Farnan from San Diego State University. Students were asked to write a short mathematics autobiography in class in which they introduced themselves and described themselves as learners of mathematics. Specifically, students were asked to describe their past experiences in school mathematics and how they preferred to learn mathematics. Two raters then independently read the autobiographies and sorted them into four piles, from lowest ability to highest ability. By sorting into an even number of piles, raters were forced to choose between high-and low-average writers rather than just a large group of average writers. Each student’s work was assigned a writing ability score: high, high-average, low-average, or low. The raters were colleagues of the teacher/researcher, both English language arts teachers. After the initial sorting, the raters were asked to try to reconcile any differences in ratings. After only one discussion, the raters came to a consensus on the relative writing ability levels of all participants in the study.

The majority of data collected for quantitative analysis was through teacher-created pre-and post-tests of content knowledge. Since these materials were directly aligned to curriculum content, they had high face and content validity. In addition, a professor of mathematics education, Dr. N. Bezuk of San Diego State University, reviewed the curriculum and tests and gave suggestions, all followed by the researcher, in order to ensure their validity. In addition, Dr. Bezuk had prior experience with the specific curriculum being used in the study. These assessments are available in Appendix B.

Student reports of the efficacy of the treatments were collected in three ways: Student writing samples, teacher field notes, and student interviews. In addition to content prompts, students were asked following each unit of study, or treatment period, to evaluate the efficacy of their writing experiences during the treatment period. Students were asked to evaluate in writing whether the prompts they responded to helped them understand the mathematics after each treatment and at the end of the study, and to make comparisons between the different types of prompts and whether or not they found them useful. The results of these written responses are described and discussed in the results section of this paper. At times, students were more willing to say how they feel about a given task than to write about their feelings, especially low-achievers in both mathematics and writing and expressed their views during the course of a lesson. Teacher field notes were kept in order to supplement the content of student written responses, both oral and written. A listing of the prompts is found in Appendix C.

At the end of the study, several students were selected from the participants for semi-structured interviews in order to gain further insight into the efficacy of the various writing tasks involved. It was hoped that students would be selected based on the results of the data analyses, following a sequential explanatory protocol. In the sequential explanatory protocol, a researcher identifies interesting and/or anomalous findings in the quantitative data related to specific participants and follows up with further data collection in order to explain those findings. However, many of the students in this study whose data were especially interesting, such as large fluctuations in either achievement or attitude gain, declined invitations to participate in subsequent interviews. As a result, all participants in the study were invited to participate in the interview process. Individual interviews were conducted by two former mathematics teachers who volunteered to help with the project. A semi-structured interview protocol was developed by the teacher for each interview participant. Since the interview process occurred several weeks after the treatment period, each interview began with a review of the different types of writing. Students were then asked to recall what they remembered most about writing in mathematics, whether or not they enjoyed the writing, what was easiest about writing in math, and whether they felt the writing helped them understand the mathematics better than doing more problems. Following the common introductory questions, students were asked about their specific responses to individual

evaluation prompts. For instance, one participant was asked to explain what they meant when they said that summary writing helped them to “get their point across.” Following questions clarifying individual responses to evaluation prompts, several questions were provided to the interviewer asking students about whether or not writing helped the student visualize the mathematics, to recognize patterns, or relationships between parts of the content, and whether writing distracted or confused the student. While the individual interviews were semi-structured, the interviewers were free to follow any train of thought or interesting path a student, or the interviewer, might want to take.

In addition to individual interviews, one group interview for each treatment group was conducted. While the individual interviews were semi-structured, the content group interviews were entirely up to the interviewers, who wanted to follow-up on some of their “wonderings”. In the group interviews, the interviewers presented the groups with a short lesson, then asked students to write about the lesson and share their thoughts during the writing process. The results of these interviews, as well as data from both the quantitative and qualitative portions of the study were triangulated in order to verify the validity of any patterns identified.

DATA COLLECTION TIMETABLES AND PROCEDURES

After gaining approval from the site principal and the Institutional Review Boards of both San Diego State University and the University of San Diego, data collection took place over one school semester in the 2006-2007 traditional calendar school year (from the beginning of September, 2006 to the end of January, 2007). The assessment of writing ability was administered at the beginning of the study and interviews took place during the spring of 2007,

The Minnesota Mathematics Attitude Inventory was administered, minus the *perception of the mathematics teacher subscale*, as a pre-and post-assessment of student attitudes towards mathematics. In addition, students were asked to respond to the eight items on the *self-concept of mathematics* subscale after the first and second rounds of treatment in order to determine if the use of specific prompts affected students’ self concept of ability for that unit of study.

The study was comprised of three treatment groups over three units of study. A unit of study was defined here as a cohesive unit of instruction around a central topic. The three topics, or units of study, were probability, an introduction to linear functions and their representations (graphing), and proportions. Groups were comprised of all students in each of three classes. Each unit of study was approximately five weeks long. In the first unit of study, Group 1 responded to procedural prompts--prompts that ask them to explain how to perform a given mathematical task. Group 2 responded to summary prompts in which students summarized their learning by identifying and explaining important points in a lesson or sequence of lessons. Group 3 responded to metacognitive prompts through which they monitored their own understanding by writing about what they did or did not understand about the content. Each treatment group took the same pre-and post-test, and the teacher/researcher was careful to provide the same instruction using the same instructional materials and activities. The same procedure was followed for the second unit of study with Group 1 responding to metacognitive prompts, Group 2 responding to procedural prompts, and Group 3 responding to summary prompts. In the third unit of study, Group 1 responded to summary prompts, Group 2 to metacognitive prompts, and Group 3 to procedural prompts, allowing the opportunity for each Group to learn about and experience each type of writing task (see Table 1).

Table 1. Rotation of Groups, Tasks, and Units of Study

GROUP	1st Unit of Study	2nd Unit of Study	3rd Unit of Study
1	Procedural	Metacognitive	Summary
2	Summary	Procedural	Metacognitive
3	Metacognitive	Summary	Procedural

Students in each group were instructed in responding to each type of prompt, including modeling, group/pair writing, and the availability of reference charts. Content writing tasks were assigned twice a week for a total of 10 per unit of study. Evaluation writing tasks through which students evaluate the efficacy of writing in mathematics were assigned at the end of each unit of study (see Appendix C).

Teacher-created pre-and post-assessments were administered for each unit of study. A complete listing of the pre-and post-tests appear in Appendix B.

Post-test data for students who were absent for and do not make up a majority of the writing tasks in a given unit of study were not included. In addition, students must have completed all practice assessments, reported their readiness for the tests, and/or attended tutoring sessions in order to prepare adequately for the assessments as determined by the teacher/researcher in order for their achievement data to be included in the study.

Teacher field notes were recorded daily, preferably after the assignment of each relevant task, as time permitted. These notes included how the prompts were actually administered, whether the writing was modeled, guided, individual, or group-written. Any oral or otherwise communicated student responses were be noted (i.e., body language, behavioral issues, etc.).

DATA ANALYSIS

Quantitative

Descriptive statistics, including measures of central tendency and dispersion, were first conducted in order to gain an overall description of the sample population. Since the main purpose of this study was to examine the effects of several variables, both independent and demographic, on two dependent variables, inferential measures were analyzed using multiple regression techniques. Many demographic variables, including gender (Millican, 1994), ethnicity (McCoy, 2005), prior achievement (Reynolds & Walberg, 1992), and socioeconomic status (McCoy, 2005), have been postulated and shown to have an effect on mathematics achievement, either directly or indirectly. For this reason, each model in this study included these demographic factors as independent variables along with general writing ability and the two variables specifically under study, the types of prompts, and self-concept of ability in mathematics. All statistical analyses were conducted through SPSS, Version 14.0.

Analysis procedures began by screening the data for outliers and missing data, and to evaluate assumptions test assumptions, such as linearity, normality, and homoscedasticity (similarly dispersed scales of continuous variables). When these assumptions were met, multiple regression analyses were conducted and results are reported through model summaries, ANOVA tables, and/or coefficients tables. Such goodness of fit measures as R^2 and standardized Beta weights were used to determine the effects of the independent

variables on the dependent variables. R^2 is reported as the coefficient of determination, the percentage of variance explained by the combined predictor variables. Beta weights are reported to indicate those variables that significantly contributed to the model, and F-ratios and p values are presented to indicate the degree to which the model predicts the dependent variables. A level of $p < .05$ was the significance criterion.

Below, each research question is presented. Dependent and independent variables are given, along with their definitions and measures.

Research Question #1: Does the type of writing task, as defined by the nature of the prompt given, have an effect on student achievement in mathematics for low achieving 8th grade students?

- Dependent variable: Change in student achievement as measured by teacher-created pre-and post unit tests.
- Independent Demographic Variables: Gender; Socioeconomic status (SES); Ethnicity
- Independent Predictor Variables: Type of prompt (procedural); Self-concept of ability as measured by the Self-Concept scale on the Mathematics Attitude Inventory (MAI); Prior achievement level as determined by the 7th grade California Standards Tests (Basic, Below Basic, and Far Below Basic); general writing ability as measured by the [to be determined].

Research Question #2: Does the type of writing task, as defined by the nature of the prompt given, have an effect on student self-concept of ability for low achieving 8th grade students?

- Dependent Variable: Change in self-concept of ability as measured by the Self-Concept scale on the Mathematics Attitude Inventory (MAI)
- Independent Demographic Variables: Gender; SES; Ethnicity
- Independent Predictor Variables: Type of prompt (Procedural); Prior achievement level as determined by the 7th grade California Standards Tests (Basic, Below Basic, and Far Below Basic); general writing ability as measured by the [to be determined].

Research Question #3: Does student self-concept of ability have an effect on student achievement for low achieving 8th grade students?

- Dependent Variable: Change in student achievement scores as measured by teacher-created pre-and post-tests during unit in which three specific types prompts were used
- Independent Demographic Variables: Gender; SES; Ethnicity
- Independent Predictor Variables: Students' self-concept of ability as measured by the MAI; Prior achievement level as determined by the 7th grade California Standards

Tests (Basic, Below Basic, and Far Below Basic); general writing ability as measured by the [to be determined].

Qualitative

Earlier studies suggested that, while quantitative results may show no significant difference between student achievement and attitudes toward mathematics before and after using a writing intervention, qualitative results often indicated important changes in some students' performance in and attitudes towards mathematics. These findings validate or contradict quantitative results on a more personal level, and give insight into why changes did or did not take place in achievement or attitude as a result of treatment conditions. For this reason, the data were analyzed using a concurrent triangulation strategy (Creswell, 2003) in which qualitative data were gathered through teacher field notes, student writing in response to evaluation prompts, and student interviews in, "an attempt to confirm, cross-validate, or corroborate findings" of the quantitative analysis of data (p. 217). The primary purpose of analysis was triangulation through coding, though a constant comparative methodology proved valuable as themes relating to achievement and self-concept of ability outside those posed in the research questions emerged. In addition, a sequential explanatory strategy (Creswell, 2003) was planned following the collection and analysis of quantitative data for each unit of study. This strategy is useful to explain or interpret surprising findings or themes that emerge during the analysis of quantitative data and is conducted sequentially, after data have been analyzed quantitatively. Unfortunately, as explained above, participants whose data were identified as surprising or interesting during quantitative analyses were reluctant to participate in the interview process and the sequential explanatory strategy was limited to clarifying student responses to evaluation prompts.

LIMITATIONS/DELIMITATIONS

Several limitations arose during the course of the study. For instance, writing and instructional time was occasionally interrupted due to conditions beyond the control of the teacher/researcher, resulting in a slight variation of writing time between groups. In addition, instruction was not completely uniform for each class due to refinement of the lesson from one class to another or to differences in the personalities and abilities of the three groups, a natural part of the educational environment. Social dynamics created some differences in

instruction and in the way tasks were carried out from class to class. Furthermore, inclusion/exclusion decisions may not have been effective, especially since writing tasks may often be group-driven and those who were absent may have had to write without the benefit of the conversation of their peers. Finally, the cumulative effects and sequence of writing tasks may affect student achievement gains and are not addressed by this study.

Delimitations included the short-term nature of the study, the frequency of writing, lack of a control group, the inclusion of low-achieving students only, and the fact that many of the students were not from the school community, increasing the probability of resistance.

CHAPTER 4

RESULTS

This study investigated the effects of various kinds of writing prompts on the achievement and attitude of struggling eighth grade students in a mathematics course designed to prepare them for algebra in ninth grade by exploring three questions:

- Does the type of writing task, as defined by the nature of the prompt given, have an effect on student achievement in mathematics for low achieving 8th grade students?
- Does the type of writing task, as defined by the nature of the prompt given, have an effect on student self-concept of ability for low achieving 8th grade students?
- Does student self-concept of ability have an effect on student achievement for low achieving 8th grade students?

Subjects came from the three sections (periods) of Middle School Algebra, a course for eighth grade students not yet ready for Algebra 1-2 as determined by their scores on the California Standards Tests (CSTs). There were five levels of proficiency on the CSTs: Advanced, Proficient, Basic, Below Basic, and Far Below Basic. Students were considered by the district in which the study took place to be prepared for algebra in eighth grade if they scored either Proficient or Advanced on the CSTs. Students who scored Below Basic or Far Below Basic on the CSTs were placed in Middle School Algebra in order to prepare them further for algebra in ninth grade. Students who scored in the lower half of the Basic CST level were also given the option of enrolling in Middle School Algebra. The course was taught by the researcher at a suburban public school in Southern California during the 2006-2007 school year. The enrollment in each section was approximately the same, ranging from 26-28 students.

Due to normal student class changes, two students entered the class after the first week of the beginning of the study and one student changed from one experimental group to another after the first treatment unit, or unit of instruction. As a result, the data from the two students who entered the class late are incomplete for the first unit and the data for the student who changed periods are incomplete for the second unit of treatment (instruction). Their data are not included in the data sets for those units. Scores for those who were missing

two or more responses to the eight-question subscale on the Minnesota Mathematics Inventory data were not calculated for that unit and are considered missing data. No evidence was present to suggest that data were missing for any other reason than completely at random. If participants submitted seven out of eight responses to the subscale, the missing item was replaced by the national mean score suggested by the publisher of the inventory. In addition, if a participant circled two adjacent responses, it was assumed the participants intended a score between the two responses available to them. This assumption was based on teacher observations during the survey. The mean of the two values was entered as the response (i.e., if participants circled 2 and 3, their response was entered as a 2.5). If participants circled two non-adjacent responses, it was assumed the participants mistakenly circled the two responses. Their score was not computed and considered missing data. A total of 81 students participated in the treatment phase of the study: 27 students in Group 1 (period 3), 26 students in Group 2 (period 5), and 28 students in Group 3 (period 7).

This study was conducted in two phases. The first phase, referred to here as the treatment phase, took place over 16 weeks in the fall semester of the 2006-2007 school year. During this phase, demographic and experimental data were collected during three five-week units of instruction (see Chapter 3 for details), as well as ongoing data in the form of researcher field notes and student evaluations of the writing they had done as a part of the regular course requirements. Data were collected about student writing ability and prior knowledge, both possible factors that previous research indicates may mediate between writing and achievement (see Chapter 2). Seventy-seven out of the 81 students in the classes attempted the majority of writing assignments during the first unit of instruction, and 78 out of 81 students attempted the majority of writing assignments during the second and third units of instruction.

The second phase, referred to here as the interview phase, took place during the last four weeks of the 2006-2007 school year following the preliminary analysis of the quantitative data. Originally, students for interviews were to be determined by the preliminary analysis of the data. Students with very large or small changes in achievement and/or self-concept of ability, or identified through teacher observation as having unique viewpoints or experiences with the writing process were identified and directly asked to participate in the interview process. However, very few of these students agreed to be

interviewed. As a result, parental consent and student assent forms to participate in interviews conducted by state-certified mathematics teachers or administrators were given to all students in each section. Of the 81 students who participated in the treatment phase of the study, 21 students and their parents consented to participate in follow-up interviews about their experiences in writing in mathematics. All of these students participated in individual interviews and 20 of them participated in group interviews. The student who did not participate in the group interview for her section was absent the day of that interview.

Data for the study were collected in several ways (see Chapter 3 for details). At the beginning of the study, students were asked to provide a writing sample in order to determine relative writing ability, and they completed a 48-item mathematics attitude inventory. Student records were searched for data regarding prior knowledge, ethnicity, and socioeconomic status. Ethnicity and prior knowledge were determined by parent declarations on admissions forms and student scores on the previous year's California Standards Tests (CSTs) respectively. Socioeconomic status was determined by lunch status and fell into three categories: (1) Free lunch; (2) Reduced lunch; (3) Assistance denied or not requested by parent.

During the study, there were three units of study, each lasting approximately five weeks. Experimental data were collected through teacher-created unit pre-and post-tests. Students were asked to write approximately twice a week for a total of ten times per unit of instruction. After each of the first two unit post-tests, students were asked to respond to the 8-item self-concept of ability subscale of the 48-item mathematics attitude inventory and to evaluate the type of writing they had done during the unit. After the third unit of instruction, students were once again given the 48-item mathematics attitude inventory. During the treatment, the researcher kept field notes on student responses to writing and other topics related to the implementation of the treatments.

Following preliminary statistical analyses of the quantitative and qualitative data collected during the first phase of the study, students were asked to participate in one-on-one and group interviews about writing in mathematics. Digital audio files were collected from each of these interviews, and teacher field notes of comments by the interviewers were also kept.

The first section of this chapter describes the results of the statistical analyses of the quantitative data collected, and the second section presents the results of the qualitative data collected through student writing, researcher field notes, and post-treatment interviews. Quantitative data analyses will be presented first through descriptive and univariate analysis by unit and group in order to get an overall picture of the data as they relate to the content and groups involved, then by research question. Multivariate analyses, specifically linear regressions, were conducted in order to explore the effects of the independent variables included in the study on the dependent variables of achievement and self-concept of ability. Linear regression was also used to explore the third research question about the relationship between achievement and self-concept of ability. Table 2 provides the full list of variables included in the study, with descriptions of each variable. Throughout, in reference to a specific student, the pronoun will be gender-specific. Otherwise, individuals will be referred to as male.

MEASUREMENT OF ACHIEVEMENT

Data related to achievement were analyzed in four ways. One way achievement gain was measured was by the difference in pre-and post-test scores. Raw scores were changed to percents, and the differences between these pre-and post-test scores were computed for this measure of achievement gain. For example, a student with a pre-test score of 20% and a post-test score of 60% made a 40% gain in achievement when measured as a percent difference. This measure was referred to as the percent change for the purposes of this study.

Another way of measuring achievement gain, called the achievement change ratio for the purposes of this study, was by finding the ratio between the students' amount of gain, as described above, divided by the amount they could have gained , or:

$$\frac{\text{Post-test score}-\text{Pre-test score}}{100-\text{Pre-test score}}$$

For example, if a student scored a 20 on his pre-test and a 60 on his post-test, the change in achievement would be 40 and the achievement change ratio would be 40/80, where 80 is the number of points he could have scored if he scored 100% on his post-test. His achievement change ratio, then, would be 0.5, or 50%, meaning he gained 50% of what he could have gained.

Table 2. Specification of Variables in the Study

Name and Type of Variable	Description	SPSS Descriptor
Gender Independent/demographic		Male=1
		Female=2
Socioeconomic Status (SES) Independent/demographic	Based on whether students receive free or reduced lunch	Free Lunch=1
		Reduced Lunch=2
		No Assistance=3
Ethnicity Independent/demographic		Latino=1
		Asian=2
		African American=3
		White=4
Writing Ability Independent	Based on writing samples obtained prior to experimental phase of study	Lowest=1
		Low=2
		High=3
		Highest=4
Prior Knowledge Independent	Based on students' seventh grade scores on the California Standards Tests	Far Below Basic=1
		Below Basic=2
		Basic=3
Type of Prompt Independent		Summary=1
		Procedural=2
		Self-Monitoring=3
Type of Unit Independent	Unit of study Unit 1–Probability Unit 2–Graphing Unit 3–Ratio and Proportion	Unit 1=1
		Unit 2=2
		Unit 3=3
Group Independent	Treatment group	Group 1=1
		Group 2=2
		Group 3=3
Achievement Dependent	Change in achievement as measured by pre-and post-tests	Percent Change
		Achievement Change Ratio
		Achievement Level
		Percent Change Ratio
Self-Concept of Ability Dependent–Question 2 Independent–Question 3	Change in self-concept of ability in mathematics as determined by pre- and post-unit scores on the self-concept of ability self-concept of ability subscale on the Minnesota Mathematics Attitude Inventory	SCA

A third method for measuring achievement gain was by the difference in achievement levels. In this measure, the items on the test were divided into four levels of achievement, labeled 1-4, with one being the lowest and four being the highest, corresponding roughly to far below basic, below basic, basic, and proficient achievement levels. For instance, there were nine items on the Unit 1 pre- and post-tests that were considered low-level skills. Each item was worth two points and, since the overall test was worth 54 points, any score up to 33% (18/54) was considered Level 1 achievement. Level 2 achievement consisted of six items, so a score between 34% and 56% (19/54 to 30/54) was considered Level 2 achievement, and so on. The levels varied from test to test according to the number of items at each level of difficulty and the total items on the test. The difference between the pre- and post-test levels was used as the measure of the achievement gain. Using the example above, a student who scored 20% on the pre-test and 40% on the post-test made a one level gain if Level 1 on that unit fell between 0% and 33%, and Level 2 on that unit fell between 34% and 56%.

Finally, achievement gain was measured by percent of increase or decrease, or the percent of change ratio, comparing the amount gained to the students' pre-test scores. This ratio is similar to the achievement change ratio and was computed as follows:

$$\frac{\text{Post-test score} - \text{Pre-test score}}{\text{Pre-test score}}$$

With this measure, the student who scored 20% on the pre-test and gained 40 percentage points between the pre- and post-test would give the student a percent of increase ratio of 40/20, or 200%, meaning the student gained 200% over what they “knew” at the beginning of the unit. In contrast, the student who scored only 1% on the pre-test and made and gained 40 percentage point during the unit would have a percent of increase ratio of 40/1, a 400% gain over what they “knew” at the beginning of the unit. Eight extreme values in the data for this measure of achievement gain affected the related statistics. However, since the scores for these extremes were valid, they were not eliminated from the data set and additional tests were run to confirm any findings related to the percent of change as needed. A comparison of trimmed means showed that outliers did not affect the statistics related to the other three measures of achievement gain.

Descriptive and Preliminary Univariate Analysis by Unit: Measures of Central Tendency

In this section, the data will be described by descriptive and preliminary inferential statistical analysis by unit. Table 3 below presents a comparison of the measures of central tendency for each of the four types of achievement gain measures by unit of study. As the data suggest, the measures of central tendency for Units 1 and 3 (probability and proportion, respectively) are similar. However, there are marked differences between these two units and Unit 2 on graphing, tables, and equations. For example, for the measure of percent difference, both the means and medians are close to the same ($M_{\text{Unit 1}}=56.24$ and $M_{\text{Unit 3}}=56.53$ and $Mdn_{\text{Unit 1}}=57.00$; $Mdn_{\text{Unit 3}}=58.00$), while they are markedly different from the mean and median of Unit 2 ($M_{\text{Unit 2}}=38.93$ and $Mdn_{\text{Unit 2}}=38.00$). This is especially interesting since many students reported to the teacher at the beginning of Unit 2 that they had studied the same curriculum from the same publisher in summer school the previous summer. This information implies that students not only had more prior knowledge going in to Unit 2 than into either Units 1 or 3, but also that they had less achievement gain to make. Therefore, it is not surprising that the achievement gain on Unit 2 was smaller in comparison with the other two units.

Table 3. Measures of Central Tendency in Achievement Gain by Unit

Measure of Achievement Change	Measure of Central Tendency	Unit 1	Unit 2	Unit 3
Percent Difference	Mean	56.24	38.93	56.53
	Median	57.00	38.00	58.00
Level	Mean	1.88	1.44	1.82
	Median	2.00	1.00	2.00
Achievement Change Ratio	Mean	.67	.50	.65
	Median	.72	.52	.67
Percent of Change	Mean	11.00	3.95	12.38
	Median	5.53	2.28	5.00

In order to compare further the differences in prior knowledge as measured by pre-test scores between the three units of study, a one-way analysis of variance (ANOVA) was conducted using the type of unit as the factor and the pre-test scores for each unit as the dependent variable. There was a statistically significant difference between the pre-test scores for each unit at the .05 significance level ($F(2,238)=11.64$; $p=.00$). However, since

Levene's test indicated that the assumption for homogeneity of variance (homoscedasticity: no significant difference between the variances in the data for each group) was not met ($F(2,238)=14.81, p=.00$), Welch's t -test was conducted to control for the inequality of variances, and the statistically significant differences between pre-test scores was confirmed ($t(2)=11.21; p=.00$). According to Cohen (as cited in Pallant, 2005), an eta squared (η^2) statistic of 0.02 indicates a small practical effect, a statistic of 0.06 indicates a medium effect, and a statistic of 0.14 indicates a large effect. In this case, the eta squared statistic was medium ($\eta^2=0.09$), indicating that the mean pre-test scores were not only statistically significant, but also had a practical effect. In addition, a post-hoc comparison using the Tukey HSD test indicated that the mean scores between the pre-tests for Units 1 and 3 were significantly different from the mean score of the pre-test for the second unit (Unit 1: $M=14.57, SD=12.04$; Unit 2: $M=20.54, SD=15.33$; Unit 3: $M=11.32, SD=8.72$). However, the means of Units 1 and 3 were not significantly different. Students, therefore, began the unit with more knowledge and/or skills on Unit 2 than on the other two units.

A one-way between-groups analysis of variance, as shown in Table 4, also verified that there were statistically significant differences at the .05 significance level in all four measures of achievement between the units ($F_{\text{PercentDiff}}(2,236)=24.60, p=.00$; $F_{\text{AchChRatio}}(2,236)=13.72, p=.00$; $F_{\text{PercentChangeRatio}}(2,235)=8.12, p=.00$; $F_{\text{AchLevel}}(2,232)=7.06, p=.00$). Like the variances in pre-test scores above, the data for the Percent of Change Ratio did not meet the assumption of homoscedasticity according to Levene's test ($F(2,235)=13.05, p=.00$), indicating that the variance for this measure of achievement change was significantly different from the variances of the other three measures. Again, to control for the inequality of variances, Welch's t -test was conducted to confirm the statistically significant differences between the units ($t(2)=14.24; p=.00$). Here, the actual differences in the mean scores between the groups ranged from medium to large. The effect for achievement as measured by percent difference was large ($\eta^2=0.17$), while the effects were medium for achievement gain by level, achievement change ratio, and percent of change ($\eta^2_{\text{Percent Change}}=0.10, \eta^2_{\text{Percent Level}}=0.06$, and $\eta^2_{\text{Ach Change Ratio}}=0.06$, respectively).

Table 4. Achievement Comparison of Means by Unit: Significance Statistics

ANOVA		Sum of Squares	df	Mean Square	F	Sig.
Achievement Percent Difference	Between Groups	15769.930	2	7884.965	24.598	.000
	Within Groups	75651.928	236	320.559		
	Total	91421.858	238			
Achievement Change Ratio	Between Groups	1.239	2	.619	13.715	.000
	Within Groups	10.657	236	.045		
	Total	11.895	238			
Achievement Percent Change	Between Groups	3200.437	2	1600.219	8.117	.000
	Within Groups	46331.240	235	197.154		
	Total	49531.677	237			
Achievement Level Change	Between Groups	9.201	2	4.600	7.062	.001
	Within Groups	151.123	232	.651		
	Total	160.323	234			

Table 5 shows that post-hoc comparisons using the Tukey HSD test indicated that the mean scores for Groups 1 and 2 were significantly different from the mean score of Unit 2 for all four measures of achievement, while the mean score of Unit 1 did not significantly differ from that of Unit 3. For each measure of achievement, the column titled "Mean Difference" indicates significant differences at the .05 level in the rows for the graphing, or second, unit, by displaying an asterisk next to the statistics for both of the other units. For example, in the cell for Achievement Percent Change, the mean difference for between achievement for the graphing and probability units is -6.99, the standard error is 2.23, and $p=.006$. In the same cell, the mean difference in achievement between graphing and proportions is -8.34, the standard error is 2.22, and $p=.001$. At the same time, no significant differences are found in achievement between the other two units.

Table 5. Comparison of Achievement Means by Unit: Tukey HSD

Dependent Variable			Mean Difference (I – J)	Std. Error	Sig.
	(I) Type of Unit	(J) Type of Unit			
Achievement Percent Change	Probability	Graphing	6.98811*	2.23429	.006
		Proportions	-1.35530	2.23429	.817
	Graphing	Probability	-6.98811*	2.23429	.006
		Proportions	-8.34341*	2.22010	.001
	Proportions	Probability	1.35530	2.23429	.817
		Graphing	8.34341*	2.22010	.001
Achievement Level Change	Probability	Graphing	.44562*	.12885	.002
		Proportions	.06260	.12966	.879
	Graphing	Probability	-.44562*	.12885	.002
		Proportions	-.38301*	.12843	.009
	Proportions	Probability	-.06260	.12966	.879
		Graphing	.38301*	.12843	.009
Achievement Percent Difference	Probability	Graphing	17.48526*	2.84899	.000
		Proportions	.54606	2.84029	.980
	Graphing	Probability	-17.48526*	2.84899	.000
		Proportions	-16.93920*	2.82215	.000
	Proportions	Probability	-.54606	2.84029	.980
		Graphing	16.93920*	2.82215	.000
Achievement Change Ratio	Probability	Graphing	.16622*	.03381	.000
		Proportions	.03169	.03371	.616
	Graphing	Probability	-.16622*	.03381	.000
		Proportions	-.13454*	.03350	.000
	Proportions	Probability	-.03169	.03371	.616
		Graphing	.13454*	.03350	.000

The data clearly show, then, that there were significant differences not only in how much students knew before Unit 2 and the other two units, but that there were significant differences in achievement gain between Units 1 and 3 and Unit 2 (Tables 4 and 5) and that achievement gains on Unit 2 were lower than on the other two units (Table 3).

Descriptive and Preliminary Univariate Analysis by Unit: Measures of Dispersion

The dispersion, or spread, of the data can be described by their range and standard deviation. Table 6 presents the range and standard deviation for each measure of

Table 6. Dispersion of Achievement Data by Unit

Measure of Achievement Change	Measure of Dispersion	Unit 1	Unit 2	Unit 3
Percent Difference	Range	82	81	89
	Standard Deviation	18.09	16.31	18.05
Level	Range	3	3	3
	Standard Deviation	.89	.69	.84
Achievement Change Ratio	Range	.89	.95	1.00
	Standard Deviation	.21	.20	.21
Percent of Change	Range	78.45	24.00	88.00
	Standard Deviation	15.23	4.56	18.78

achievement. An examination of the ranges in the data by unit shows that the third unit had a slightly higher range of scores when measured as percent, ($\text{range}_{\text{Unit 1}}=82$; $\text{range}_{\text{Unit 2}}=81$; $\text{range}_{\text{Unit 3}}=89$), and by change ratio ($\text{range}_{\text{Unit 1}}=.89$; $\text{range}_{\text{Unit 2}}=.95$; $\text{range}_{\text{Unit 3}}=1.00$), indicating that the unit may have been less familiar to students at the beginning so that they were able to make greater gains on average. The range of data related to the percent of change ($\text{range}_{\text{Unit 1}}=78.45$; $\text{range}_{\text{Unit 2}}=24.00$; $\text{range}_{\text{Unit 3}}=88.00$), on the other hand, revealed that students may have begun Unit 2 on graphing with more prior knowledge and, therefore, had less gain to make.

As seen in Table 6, the range and standard deviation of the data were much lower for Unit 2 when achievement was measured by the percent of change ($\text{range}_{\text{Unit 1}}=78.45$; $\text{range}_{\text{Unit 2}}=24.00$; $\text{range}_{\text{Unit 3}}=88.00$). For this measure, the data were less spread out than for the other measures. The range of the data was somewhat wider when measured by the percent difference, the achievement change ratio, and the percent of change. The standard deviations for Units 1 and 3 were more similar in all measures than for Unit 2.

The skewness and kurtosis of a distribution is a measure of the degree of asymmetry of the data. In a normal distribution, the data are equally distributed around the mean, or symmetrical. In a positively skewed distribution, the data are “clumped” towards the lower end of the score continuum and a tail formed by a small percentage of the scores is strung out across the upper end of the score continuum. In a negatively skewed distribution, the data are “clumped” towards the upper end of the score continuum and a tail formed by a small percentage of the scores is strung out across the lower end of the score continuum. Kurtosis refers to how “peaked” the data are. A normal distribution is mesokurtic, neither overly

peaked or overly flat. A leptokurtic distribution is overly peaked, and a platykurtic distribution is overly flat. The shape of the distribution is important because many statistical tests assume that the distribution of scores in a data set is normal. If those assumptions are not met, results may be incorrect, invalid, and not meaningful.

As shown in Table 7, skewness was negative in each unit for achievement level change and the change ratio, showing a greater percentage of the data toward the upper end of the score continuum on these measures of achievement. In other words, on these two measures of achievement, more students' achievement scores were above the mean than below it. When measured by percent difference, Units 1 and 3 distributions were negative, but slightly positive for Unit 2. The distributions of the data when measured as a percent of change, however, were highly positively skewed for all three units. The skewness values for the first three measures of achievement all fall between -1.0 and 1.0, values defined by several authors (Huck, 2008) as well within the parameters considered normal distribution for the purposes of multivariate parametric analysis. The same conditions occurred with kurtosis, with the values indicating relatively normal distributions (between -1.0 and 2.0) for all measures of achievement except percent of change.

Table 7. Skewness of Achievement Data by Unit

Measure of Achievement Change	Measure of Distribution	Unit 1	Unit 2	Unit 3
Percent Difference	Skewness	-.525	.008	-.687
	Kurtosis	-.072	-.173	.845
Level	Skewness	-.653	-.118	-.408
	Kurtosis	-.056	-.155	-.250
Achievement Change Ratio	Skewness	-.926	-.353	-.742
	Kurtosis	.299	-.262	.341
Percent of Change	Skewness	2.929	2.401	3.207
	Kurtosis	9.000	6.072	10.613

In summary, the descriptive statistics by unit showed that Unit 2 on graphing, tables, and equations was different in several ways from the other two units on probability and proportions. Students had, on average, more prior knowledge of the content for this unit compared with the other units, yet the mean achievement scores were statistically lower than on the other units for all measures of achievement chain at the .05 significance level. In addition, the spread of the data was narrower for Unit 2 according to measures of range and

standard deviation on all three measures of achievement gain. The shapes of the distributions were relatively normal except for the percent of change measure. Again, Unit 2 stood out as having the most positive skewness on each of the relatively normal distributions, indicating that more student achievement scores tended to be below the mean for Unit 2 than for the other two units. The next section will explore the data by group using descriptive and preliminary inferential statistics.

Descriptive and Preliminary Inferential Statistics by Group

FREQUENCIES

Descriptive statistics were examined in order to determine if the experimental groups of participants, the three classes taught by the teacher/researcher, were different in any significant way with regard to their academic achievement throughout the unit. A general description of the groups may be found in Chapter 3. An examination of demographic variables revealed that there were some demographic differences between groups (see Table 8). For instance, Groups 1 and 2 had approximately the same number of males (52%) as females (48%), while Group 3 had more females (68%) than males (32%). By ethnicity, Group 1 differed from the other two groups. Only 52% of Group 1 was Latino, compared with 81% in Group 2 and 79% in Group 3. In addition, Group 1 also had more African American students (22%) compared to Groups 2 and 3 (4% and 11%, respectively). Group 1 also differed from the other two groups in that it had far fewer students who received reduced or free lunch (33%) and reduced lunch (11%) compared to Groups 2 (free lunch–58%; reduced lunch–19%) and 3 (free lunch–68%; reduced lunch–14%). These differences in demographic variables indicate a need to control for their effects in subsequent linear regressions.

Table 8. Demographic Differences Between Experimental Groups

	Gender		Ethnicity				Socioeconomic Status		
	M	F	Latino	White	Asian	African Am.	Free	Reduced	No Assist.
Group 1	52%	48%	52%	26%		22%	33%	11%	56%
Group 2	50%	50%	81%	12%	4%	4%	58%	19%	23%
Group 3	32%	68%	79%	11%		11%	68%	14%	18%

Group 3 also differed from the other two academically, with only 7% of the students in that class scoring Basic on the California Standards Tests, while Groups 1 and 2 had three times as many students who scored Basic on the same tests (22% and 23%, respectively). Since there appeared to be academic differences among the groups, a one-way analysis of variance (ANOVA) was conducted using the group as the factor and the pre-test scores for each group as the dependent variable. The results, presented in Table 9, indicated a significant difference in mean pre-test scores, a measure of immediate prior knowledge, between groups ($F(2,238)=4.59$; $p=.011$). The Tukey HSD post hoc test revealed a statistically significant difference between the mean pre-test scores of Group 3 and those of Groups 1 and 2, with Group 3 scoring significantly lower on average than the other two groups. There was no significant difference between the average pre-test scores of Groups 1 and 2 (see Table 10), suggesting that Group 3 had less prior knowledge than the other two groups. The effect size, the practical difference between the pre-test scores of the groups was, however, small (0.04).

Table 9. Comparison of Pre-Test Scores by Group

ANOVA Pre-Test Score	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1475.074	2	737.537	4.585	.011
Within Groups	38283.126	238	160.853		
Total	39758.199	240			

Table 10. Comparison of Means in Prior Knowledge by Group: Tukey HSD

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.
1.00	2.00	.06891	2.01814	.999
	3.00	5.24021(*)	1.98712	.024
2.00	1.00	-.06891	2.01814	.999
	3.00	5.17130(*)	2.00005	.028
3.00	1.00	-5.24021(*)	1.98712	.024
	2.00	-5.17130(*)	2.00005	.028

* The mean difference is significant at the .05 level.

In addition to the variables examined above, there were significant differences in writing ability between groups ($F(2,234)=4.27$; $p=.02$). However, the effect size was small ($\eta^2=0.04$).

The results of these tests indicate the need to control for pre-test scores and writing ability on subsequent linear regressions.

MEASURES OF CENTRAL TENDENCY

Table 11 presents a comparison of the measures of central tendency for each of the four types of achievement gain measures by group. A comparison of means to trimmed means indicated that outliers and extreme values did not significantly affect the means on the data related to three out of the four measures of achievement. As found above in the analysis of data by unit, extreme values may have had an effect on the means as measured by the percent of change. However, the extreme values in the data were valid scores and were left in the data set.

Table 11. Measures of Central Tendency in Achievement Gain by Group

Measure of Achievement Change	Measure of Central Tendency	Group 1	Group 2	Group 3
Percent Difference	Mean	52.5	46.4	52.1
	Median	54	50.0	53.5
Level	Mean	1.8	1.6	1.7
	Median	2.0	2.0	2.0
Achievement Change Ratio	Mean	.64	.57	.60
	Median	.68	.60	.62
Percent of Change	Mean	7.6	6.4	12.9
	Median	4.1	3.1	3.1

A one-way between-groups analysis of variance indicated there were no statistically significant differences on average at the .05 significance level on all four measures of achievement between the groups, except for Percent of Change (Percent Difference: $F(2,236)=2.4$; $p=.092$; Achievement Change Ratio: $F(2,236)=2.0$, $p=.132$; Level Change: $(F(2, 232)=1.4$, $p=.247$; Percent of Change: $F(2,235)=4.7$, $p=.01$). Since the Percent of Change measure again failed Levene's test for homogeneity ($F(2,235)=7.0$, $p=.001$), Welch's t -test confirmed ($t(2)=3.6$; $p=.030$) that the differences in the mean achievement of Group 3, as measured by percent of change, was indeed significantly different from that of the other two groups at the $p<.05$ level, and that there was no significant difference between Groups 1 and 2 in mean achievement as measured by percent of change. The data indicate that there

were statistically significant differences between the groups with regard to academic preparation and achievement gain, especially between Group 3 and the other two groups.

MEASURES OF DISPERSION

Table 12 presents the dispersion of the data for all four measures of achievement. The data show a slight difference between the groups on some measures of achievement gain.

Table 12. Dispersion of Achievement Data by Group

Measure of Achievement Change	Measure of Dispersion	Group 1	Group 2	Group 3
Percent Difference	Range	93	85	78
	Standard Deviation	20.5	18.5	19
Level	Range	3	3	3
	Standard Deviation	.83	.84	.80
Achievement Change Ratio	Range	1	1	.89
	Standard Deviation	.24	.22	.21
Percent of Change	Range	75	58	87
	Standard Deviation	11.4	9.9	19

The most marked difference between the groups is on the percent of change measure, with Group 2 showing a much smaller range ($\text{range}_{\text{Group 2}}=58$) and standard deviation ($SD_{\text{Group 2}}=9.9$) than the other two groups ($\text{range}_{\text{Group 1}}=75$; $\text{range}_{\text{Group 3}}=87$ and $SD_{\text{Group 1}}=11.4$; $SD_{\text{Group 3}}=19$), indicating that the data were more tightly packed for this group than for the other groups. Also, a marked difference is seen between Groups 1 and 3 on both the percent difference measure and that of percent of change, with the former indicating more achievement gain by Group 1 and the latter indicating more gain by Group 3. Since the four measures of achievement gain were included in this study in order to see achievement gain from different perspectives, it is not surprising that two different measures might offer two different conclusions. Whereas Group 1 appears to have made more raw gain than the other groups, Group 3 may have made more gain relative to where they started than the other two groups.

Table 13 presents the skewness statistics of the achievement data by group. The shapes of these distributions of the data on three of the four measures of achievement gain by group are somewhat negatively skewed and platykurtic, but fall well within the parameters of what is considered a normal distribution. These distributions indicate that student

achievement scores, on average, fell somewhat above the mean and that the shape of the data is somewhat flatter than a normal curve. The percent of change measure, however, continues to yield highly negatively skewed and leptokurtic (peaked) distributions, indicating that, by this measure of achievement, most students' scores fell well below the mean. This finding indicates that students did not make high achievement gains relative to their pre-test scores even though they may have made noticeable gains in raw scores and relative to what they could have gained. The percent of change measure also indicates that for Group 3, more of the scores fell closer to the mean than in the other two groups and that fewer scores were tightly packed together. This finding is supported by the fact that the standard deviation of Group 3 for the percent of change measure was much higher than for the other two groups (see Table 12).

Table 13. Skewness of Achievement Data by Group

Measure of Achievement Change	Measure of Distribution	Unit 1	Unit 2	Unit 3
Percent Difference	Skewness	-.439	-.317	-.186
	Kurtosis	-.249	-.064	-.734
Level	Skewness	-.231	-.223	-.278
	Kurtosis	-.549	-.467	-.257
Achievement Change Ratio	Skewness	-.636	-.556	-.424
	Kurtosis	-.216	-.129	-.611
Percent of Change	Skewness	3.899	3.281	2.852
	Kurtosis	18.271	12.021	8.069

In summary, analysis of pre-test and achievement data by group shows some significant differences between the groups, especially between Group 3 and the other two groups. Group 3 had proportionally more females than males than either Group 1 or Group 2 and proportionally more Latinos than Group 1. Group 3 also was less academically prepared than Group 1 with regard to prior knowledge, and less academically prepared than either group with regard to writing ability and the pre-tests used in the study. However, results showed that, while there were no significant differences on three out of four measures of achievement gain used in the study, Group 3 did make significantly more achievement gain relative to their pre-test scores than the other two groups. This is an indicator that multivariate analysis, such as linear regression, needs to be examined in order to control for

confounders to see the effect of the groups after adjusting for all of the independent variables present in this study.

Since each treatment group experienced a specific sequence of writing prompts (summary/self-monitoring/procedural; procedural/summary/self-monitoring; and self-monitoring/procedural/summary) that may have had affected their achievement, the results of the comparison of means by group also indicated that there were no statistically significant associations between the sequence of writing and achievement scores in this study. It must be noted, however, that only three out of the six possible sequences given three types of prompts were present.

The next section of this data presentation will analyze and explore the data relative to the first research question: “Does the type of writing prompt affect achievement of eighth grade students struggling in mathematics?”

Research Question #1: Does the Type of Prompt Affect Achievement?

An examination of the dispersion of the data revealed differences in achievement gain when grouped by type of prompt (see Table 14). For the percent difference measure, the range of the data for achievement change scores was far wider following summary writing (91) than for procedural (74), with achievement change following metacognitive writing (85) between the two extremes. However, the standard deviations of the data were similar among summary, procedural, and self-monitoring prompts (19.7, 17.5, and 20.3, respectively). For the achievement change ratio measure, the standard deviations were, again, similar ($SD_{\text{summary}}=0.22, SD_{\text{procedural}}=0.21, SD_{\text{self-monitoring}}=0.23$), but the range for the achievement data related to the summary prompt had a higher range (0.98), while the range of the data related to the other two prompts was similar (0.87 and 0.89). Of necessity, the range of level change for all three prompts was three, since students could only advance three, at most. The standard deviation for this measure, however, indicated that the data related to self-monitoring were somewhat more spread, with a standard deviation of 0.90 compared to 0.79 and 0.78 for summary and procedural prompts respectively. The range of the data for the percent of change measure was similar to that of the percent change data, with the widest range in the data for the summary prompt (87.9), the lowest the procedural prompt (57.5), and the range of the data for self-monitoring somewhere in the middle (79.0). The standard

Table 14. Dispersion of Achievement Data by Prompt

Measure of Achievement Change	Measure of Dispersion	Summary	Procedural	Self-Monitoring
Percent Difference	Range	91	74	85
	Standard Deviation	19.73	17.52	20.35
Level	Range	3	3	3
	Standard Deviation	.79	.78	.91
Achievement Change Ratio	Range	.98	.87	.95
	Standard Deviation	.22	.21	.23
Percent of Change	Range	87.86	57.45	79.00
	Standard Deviation	19.41	11.05	11.53

deviation for the percent of change measure was much higher (19.4) than for the other two prompts (11.1 and 11.5), indicating very different shapes in the data related to the percent of change measure and that of the other three measures. The analysis of the dispersion of the data, then, indicates very different shapes of the data depending on which measure of achievement is used.

An examination of the skewness and kurtosis of the data shows that, again, for three of the four measures of achievement change, the data fall within the parameters of what is considered a normal distribution (see Table 15). The data for each type of prompt again fall between the -1.0 and 1.0 boundaries for skewness and -1.0 and 2.0 for kurtosis, though only the data related to summary ($p=.200$) and procedural ($p=.200$) writing, as measured by the percent difference and the data related to summary ($p=.175$) writing, were confirmed as within normal distribution limits according to the Kolmogorov-Smirnov test of normality. Normality of the data for achievement change was not confirmed by the Kolmogorov-Smirnov test for any other prompt or type of measure. Since negative skewness indicates that most achievement scores fell somewhat over the mean on all except the percent of change measure, more students scored above the mean than below it on all except the percent of change measure. Kurtosis values indicate slight to moderate flattening of the data compared to a normal distribution.

A comparison of means to trimmed means indicated that outliers and extreme values did not significantly affect the means on the data related to three out of the four measures of achievement. As found above in the analysis of data by unit and by groups, extreme values may have had affected the means, especially as measured by the percent of change. However,

Table 15. Skewness of Achievement Data by Prompt

Measure of Achievement Change	Measure of Distribution	Summary	Procedural	Self-Monitoring
Percent Difference	Skewness	-.230	-.120	-.456
	Kurtosis	-.354	-.486	-.391
Level	Skewness	-.288	-.230	-.144
	Kurtosis	-.244	-.235	-.730
Achievement Change Ratio	Skewness	-.548	-.344	-.592
	Kurtosis	-.178	-.488	-.333
Percent of Change	Skewness	3.044	3.200	3.491
	Kurtosis	8.822	10.643	17.459

the extreme values in the data were valid scores and were left in the data set. Means and medians for each group are very similar, except for the percent of change measure, supporting the conclusion that the dispersion of the data for the other three measures of achievement approaches normal distribution.

A visual examination of the means as shown in Table 16 showed that achievement following summary writing was generally higher than achievement following the other two prompts, with achievement following metacognitive writing somewhat higher than that following procedural writing on the Achievement Change Ratio and the Percent of Change measures. However, the results of a one-way ANOVA with the four types of achievement gain measure as dependent variables and the type of prompt as the factor indicated no significant differences at the .05 significance level in achievement gain between the types of prompts.

Table 16. Measures of Central Tendency in Achievement Gain by Prompt

Measure of Achievement Change	Measure of Central Tendency	Summary	Procedural	Self-Monitoring
Percent Difference	Mean	53.5	49.0	48.7
	Median	53.5	50.0	53
Level	Mean	1.8	1.7	1.6
	Median	2.0	2.0	2.0
Achievement Change Ratio	Mean	.64	.58	.58
	Median	.67	.61	.62
Percent of Change	Mean	11.2	7.2	8.7
	Median	4.2	3.6	4.6

However, the results of a one-way ANOVA with the four types of achievement gain measure as dependent variables and the type of prompt as the factor indicated no significant differences at the .05 significance level in achievement gain between the type of prompts (see Table 17).

Table 17. Comparison of Means in Achievement by Prompt

ANOVA		Sum of Squares	df	Mean Square	F	Sig.
Achievement Percent Difference	Between Groups	1013.758	2	506.879	1.323	.268
	Within Groups	90408.099	236	383.085		
	Total	91421.858	238			
Achievement Change Ratio	Between Groups	.171	2	.085	1.717	.182
	Within Groups	11.725	236	.050		
	Total	11.895	238			
Achievement Percent Change	Between Groups	554.457	2	277.229	1.330	.266
	Within Groups	48977.219	235	208.414		
	Total	49531.677	237			
Achievement Level Change	Between Groups	1.201	2	.601	.876	.418
	Within Groups	159.122	232	.686		
	Total	160.323	234			

In summary, though there were some differences between achievement scores following the different types of prompts by unit and by group, no prompt was significantly associated statistically with a change in achievement gain.

REGRESSION ANALYSES

The primary purpose of this study was to explore the relationship, if any, between the type of writing as a part of instruction in mathematics, and subsequent achievement, and the relationship between achievement and attitude, especially self-concept of ability in mathematics. While preliminary analysis showed no statistically significant relationship between the type of prompt and achievement, and only small positive relationships between achievement and attitude, it is important to explore what factors do explain differences in achievement and attitude in mathematics. Toward that end, multiple linear regression was

used to build a model of explanatory variables that best explained the variance in achievement.

Simultaneous multiple regressions were conducted for each of the four types of achievement (percent difference, achievement change ratio, percent of change, and achievement level change) as the dependent variable. Independent variables included pre-test achievement scores, self-concept of ability pre-test scores, change in self-concept of ability, prior knowledge, writing ability, and the demographic variables: ethnicity, gender, socioeconomic status, types of prompt, type of unit, and group. Stepwise regressions were then run to confirm and expand the findings of the simultaneous regressions. Prior to running linear regressions on the data, the independent variables were recoded into dummy variables. Table 18 presents the recoded variables.

Table 18. Dummy Variables

Variable	Recoding
Gender	Female=0; male=1
Latino	0 if other; 1 if Latino
African American	0 if other; 1 if African American
Asian or White	0 if other; 1 if Asian or White - omitted variable
Free Lunch	0 if other; 1 if Free Lunch
Reduced Lunch	0 if other; 1 if Reduced Lunch
No Assistance	0 if other; 1 if No Assistance - omitted variable
Lowest Writing Ability	0 if other; 1 if Lowest
Low Writing Ability	0 if other; 1 if Low
High Writing Ability	0 if other; 1 if High
Highest writing Ability	0 if other; 1 if Highest - omitted variable
Far Below Basic (Prior Knowledge)	0 if other; 1 if Far Below Basic
Below Basic (Prior Knowledge)	0 if other; 1 if Below Basic
Basic (Prior Knowledge)	0 if other; 1 if Basic - omitted variable
Unit 1 - Probability	0 if other; 1 if Unit 1
Unit 2-Graphing	0 if other; 1 if Unit 2-omitted variable
Unit 3-Ratio and Proportion	0 if other; 1 if Unit 3
Prompt 1 - Summary	0 if other; 1 if Prompt 1
Prompt 2 - Procedural	0 if other; 1 if Prompt 2
Prompt 3-Self-monitoring	0 if other; 1 if Prompt 3-omitted variable

Four separate simultaneous regressions, one each for the four measures of achievement, were run in order to explore the variables that affected achievement in this study. All output was reviewed for problems with co-linearity by examining the Tolerance

and Variance inflation factor (VIF). Variables with values less than 0.10 for Tolerance and over 30 for the VIF were removed from the analysis, as suggested by Pallant (2005). Correlations were also examined for values over 0.7 or below -0.7 and variables with these correlation coefficients were removed from analysis. Assumptions of normality were checked by examination of the Normal Probability Plot of Regression for each test run. Assumptions of normality were violated for the percent of change measure, so the statistics and findings for this measure were not included in these results. In addition, the scatter plot of the standardized residuals were all roughly rectangularly distributed, and one outlier close to 3 or -3 was found for some tests. Given the magnitude of n for all tests, this outlier was not considered problematic. Finally, Cook's Distance was examined for each test run and no value over 1 was found.

Table 19 presents a comparison of the most informative statistics contained in the models found to be the most explanatory, with SCA representing the phrase "self-concept of ability." The models with the most explanatory power were based on the percent difference ($R^2=.429$) and the achievement change ratio ($R^2=.403$), each explaining approximately 40% of the variance in students' achievement scores, and all three models were significant at .05 level. All three models shared three factors that made unique contributions to the model: lowest prior knowledge (Far Below); change in self-concept of ability; and self-concept of ability pre-test score. Not surprisingly, the self-concept of ability pre-test score and change in self-concept of ability were positively associated with achievement, while the lowest prior knowledge level was negatively associated with achievement. Only the percent difference model included the achievement pre-test score, which was negatively associated with achievement ($\beta = -.432, p = .000$) and was the greatest unique factor associated with achievement change in that model. Controlling for all other variables in the model, every 1 point gain in pre-test score was associated with a decrease in achievement as measured by the percent difference. In addition, this model was the only one to include a negative association between gender (being male) and achievement ($\beta = -.143, p = .032$). In other words, controlling for all the other variables in the model, males were associated with a .143 decrease in achievement as measured by percent difference. Not surprisingly, the lowest level of prior knowledge, Far Below Basic, made the strongest unique contribution in the other two models, and was the second strongest unique contributor in the percent difference

Table 19. Achievement Simultaneous Regressions

Measure of Achievement	R ²	Sig.	Variable	Beta	Sig.
Percent Difference	.429	.000	Prior Knowledge-Far Below Basic	-.383	.000
			Change in Self-Concept of Ability	.251	.000
			Unit Pre-Test Score	-.432	.000
			Self-Concept of Ability Pre-test Score	.278	.001
			Unit 1 - Probability	.343	.000
			Unit 3 - Proportions	.236	.001
			Gender (male)	-.143	.032
Achievement Change Ratio	.403	.000	Prior Knowledge-Far Below Basic	-.401	.000
			Change in Self-Concept of Ability	.250	.000
			Self-Concept of Ability Pre-test Score	.253	.002
			Unit 1 - Probability	.363	.000
			Unit 3 - Proportions	.262	.000
			Prompt - Summary	.134	.043
Achievement Level Change	.276	.000	SES – Free Lunch	-.201	.032
			Prior Knowledge-Far Below Basic	-.344	.001
			Unit 3 - Proportions	.260	.001
			Self-Concept of Ability Pre-test Score	.241	.009
			Change in Self-Concept of Ability	.206	.009

model. Interestingly, the first and third unit had a positive association with achievement using both the percent difference and achievement change ratio measures.

In answer to the first research question, the type of prompt found to have a significant effect on achievement was found in the achievement change ratio measure ($\beta=.134$, $p=.043$). Controlling for prior knowledge, change in self-concept of ability, self-concept of ability pre-test score, and Units 1 and 3, prompt 1(summaries) was associated with a .134 gain in achievement as measured by the achievement change ratio. It is important to note, however, that the type of prompt was not found to have a significant effect on achievement for any of the other measures of achievement.

The next section will present the results of descriptive and preliminary univariate statistics related to self-concept of ability in mathematics, followed by linear regressions to determine the effects of the type of prompt on self-concept of ability.

DESCRIPTIVE AND PRELIMINARY UNIVARIATE STATISTICS FOR SELF-CONCEPT OF ABILITY

Self-concept of ability was measured by the subscale of the same name on four administrations of the Minnesota Mathematics Attitude Inventory. This construct was measured at the beginning of the study and after each of the three instructional units. This section will explore the statistics related to self-concept of ability by unit and by group in order to get a general overview of the data. Each comparison will begin with an analysis of the pre-treatment data, followed by the analysis of data of the changes that occurred during treatment.

A visual comparison of the dispersion of the data related to self-concept of ability prior to treatment indicated some differences in self-concept of ability before each unit with an upward trend in the changes from Unit 1 to Unit 3 ($\text{range}_{\text{unit 1}}=2.38$; $\text{range}_{\text{unit 2}}=2.63$; $\text{range}_{\text{unit 3}}=2.75$), indicating a gradual increase in the spread of the data. The same trend occurred in the standard deviations ($SD_{\text{unit 1}}=0.47$; $SD_{\text{unit 2}}=0.56$; $SD_{\text{unit 3}}=0.58$), confirming the gradual spread in scores from beginning to end of treatment. Skewness and kurtosis values fell within normal distribution parameters.

A visual examination of the means of the pre-test scores for self-concept of ability as shown in Table 20 also indicates a gradual rise from beginning to end of treatment.

Table 20. Measures of Central Tendency for Self-Concept of Ability Pre-Test Scores

	Unit 1	Unit 2	Unit 3
Mean	2.35	2.69	2.76
Median	2.38	2.75	2.75

A comparison of means and medians, as well as means to 5% trimmed means indicated no effect of extreme values on the data. A one-way ANOVA was conducted in order to determine if there were significant differences in the means of the pre-test scores prior to treatment between units. Results, shown in Table 21, indicate a significant difference in self-concept of ability at the .05 significance level between the first unit and the other two units, but not between Units 2 and 3.

Table 21. Comparison of Means in Self-Concept of Ability Prior to Treatment by Unit: Tukey HSD

(I) Type of Unit	(J) Type of Unit	Mean Difference (I - J)	Std. Error	Sig.	95% Confidence Interval	
					Upper Bound	Lower Bound
Probability	Graphing	-.34146*	.08858	.000	-.5505	-.1325
	Proportions	-.41060*	.08773	.000	-.6176	-.2306
Graphing	Probability	.34146*	.08858	.000	.1325	.5505
	Proportions	-.06914	.08743	.709	-.2754	.1371
Proportions	Probability	.41060*	.08773	.000	.2036	.6176
	Graphing	.06914	.08743	.709	-.1371	.2754

* The mean difference is significant at the .05 level

In summary, there was an upward trend in self-concept of ability prior to each treatment, or unit of study. In addition, the data for self-concept of ability tended to disperse more throughout the study, perhaps indicating that students who began at nearly the same level of self-concept of ability began to gradually grow apart. There were also significant differences in self-concept of ability prior to Unit 1 and Units 2 and 3, but no significant difference in self-concept of ability prior to the second and third units. The data for the differences, or change, in self-concept of ability will next be analyzed by unit.

A comparison of the ranges of the data indicated that the change in self-concept of ability following the third unit (proportions) may have differed somewhat from that of the other two units (probability and graphing/table/equations respectively). The range for the third unit was slightly lower (range=2.75) than those of Unit 1 (range=3.25) and Unit 2 (range=3.38), indicating the scores for Unit 3 were somewhat more tightly packed. The

standard deviations for all three units, however, were nearly the same ($SD_{unit1}=0.50$; $SD_{unit2}=0.49$; $SD_{unit3}=0.49$).

Skewness and kurtosis statistics indicate that the data for change in self-concept of ability did not fall within normal distribution limits for the first two units, except for the data related to the third unit. The data after Unit 1 were highly negatively skewed (-1.767), indicating that the scores for students' self-concept of ability after Unit 1 were mostly above the above mean, and leptokurtic (7.409), or highly peaked. The scores after Unit 2 were highly positively skewed with scores mostly below the mean (1.742), and, again, leptokurtic (6.954). The skewness and kurtosis levels for Unit 3 (.365 and 1.196) were within normal distribution parameters and indicated that many of the students' self-concept of ability scores after this unit were below the mean. The results of the Kolmogorov-Smirnov test of normality confirmed that the data for Units 1 and 2 did not fall within normal distribution and limits while the data for Unit 3 did (Unit 1: $p=.000$; Unit 2: $p=.000$; Unit 3: $p=.053$).

A visual comparison of the means of the data related to differences in self-concept of ability for each unit indicated a downward trend in self-concept of ability from the first to the last unit (Unit 1: $M=0.33$; Unit 2: $M=0.10$; Unit 3: $M=-0.11$). The medians were also varied and suggested further analysis (Unit 1: $Mdn=0.37$; Unit 2: $Mdn=0.00$; Unit 3: $Mdn=-0.12$). A comparison of means to trimmed means indicated the data for Unit 2 may have been affected by extreme values ($M=0.10$ to trimmed mean= 0.33). As before, the scores were determined to be valid and left in the data set. Since the distribution of the data was primarily not within normal distribution parameters, and there was one sample of subjects measured on the same scale under three different conditions, the non-parametric Friedman two-way analysis of variance by ranks was conducted to compare the medians of the data for three units. The results of the Friedman test indicate that there was a statistically significant difference between the medians of the data related to self-concept of ability for the three units, $X^2(2)=26.36$, $p<.05$.

In summary, analysis of the data related to pre-test scores of self-concept of ability by unit indicated a significant difference between the pre-test scores prior to Unit 1 and those of Units 2 and 3, after the treatment began. In addition, analysis of the data for the differences in self-concept of ability after treatment began revealed that the data for two out of three units did not fall within a normal distribution and that there was a statistically significant

difference between the medians of the scores for the three units. Also, there was a distinct downward trend in the means and medians of self-concept of ability scores from Unit 1 through Unit 3.

Data related to self-concept of ability were analyzed next by group. A comparison of the dispersion of the data related to change in self-concept of ability prior to treatment indicates only small differences in range ($\text{range}_{\text{group1}}=2.75$; $\text{range}_{\text{group2}}=2.25$; $\text{range}_{\text{group3}}=2.50$), but a larger difference in standard deviation between Group 1 ($SD=0.62$) and the other two groups ($SD_{\text{group2}}=0.52$; $SD_{\text{group3}}=0.55$). These differences indicate there may have been a substantial difference between Group 1 and the other two groups, but not between Groups 2 and 3 prior to treatment. Group 1 seems to have had more diversity in self-concept of ability prior to treatment than the other two groups.

Visual analysis of the means of the groups related to self-concept of ability from the pre-test prior to the treatment phase indicated that there was little difference in self-concept of ability among groups (Group 1: $M=2.69$; Group 2: $M=2.51$; Group 3: $M=2.62$). Tests of normality for pre-treatment self-concept of ability indicate data for all groups fell within normal distribution parameters. Not surprisingly, the results of a one-way ANOVA with self-concept of ability as the dependent variable and the groups as the factor indicate no significant differences at the .05 significance level between the means of the groups prior to treatment.

A visual analysis of the data during treatment related to the differences in self-concept of ability during the treatment phase indicated some differences between groups. A comparison of means to trimmed means of the differences in self-concept of ability indicated that extreme values may have had some effect on the means, especially for Group 2 ($M=.67$, $Mdn=.76$) and Group 2 ($M=.63$, $Mdn=.51$). In addition, Group 1 had, on average, a somewhat larger mean self-concept of ability score after the study began. Since the data for Group 2 did not fall within the limits of a normal distribution after the treatment began, the Kruskal-Wallis Test for one continuous dependent variable and three or more categorical independent variables was conducted to determine if there was a significant difference between the groups during the treatment phase of the study. The results of the Kruskal-Wallis test indicated no significant difference in self-concept of ability between groups during the treatment phase of the study ($H(2)=2.644$, $p=.267$)

Visual examination of the data for pre-test scores in self-concept of ability by demographics show that all distributions fell within normal parameters, and comparisons of means to 5% trimmed means indicate that extreme values did not affect the means. Slight differences in range can be found in all demographic groups, though none were considered large enough to present or display here.

A visual comparison of the means of pre-test scores for self-concept of ability in mathematics indicated a difference between males ($M=22.7$) and females ($M=19.4$). A one-way ANOVA indicated a statistically significant difference between the pre-test scores of males and females ($F(1,225)$, $p=.000$). The effect size was large (0.14), indicating not only significant differences between genders on the pre-test self-concept of ability scores, but also as practical difference. Visual examination of the means between ethnicities also indicated some differences in mean pre-test scores ($M_{\text{Latino}}=20.3$; $M_{\text{Asian}}=22.7$; $M_{\text{White}}=23.4$; $M_{\text{African American}}=20.7$). Care must be taken when attaching significance to these results, since there was only one Asian participant and there were only 13 White participants in the sample. The results of a one-way ANOVA indicated that there was a statistically significant difference in the pre-test scores for self-concept of ability between the groups ($F(3,223)$, $p=.001$). The Tukey HSD test results found a significant difference between the self-concept of ability of Latino and White students, but no differences between other groups in self-concept of ability prior to treatment (see Table 22). Latino students, in other words, appear to have had a lower self-concept of ability compared to White students prior to treatment.

Table 22. Comparison of Means of Self-Concept of Ability Prior to Treatment by Ethnicity: Tukey HSD

(I) Ethnicity	(J) Ethnicity	Mean Difference (I - J)	Std. Error	Sig.	95% Confidence Interval	
					Upper Bound	Lower Bound
Latino	Asian	-2.47723	2.56247	.768	-9.1102	4.1558
	White	-3.25793*	.79311	.000	-5.3109	-1.2050
	African American	-.49056	.94533	.954	-2.9376	1.9564
Asian	Latino	2.47723	2.56247	.768	-4.1558	9.1102
	White	-.78070	2.63724	.991	-7.6072	6.0458
	African American	1.98667	2.68695	.881	-4.9685	8.9419
White	Latino	3.25793*	.79311	.000	1.2050	5.3109
	Asian	.78070	2.63724	.991	-6.0458	7.6072
	African American	2.76737	1.13245	.072	-.1640	5.6987
African American	Latino	.49056	.94533	.954	-1.9564	2.9376
	Asian	-1.98667	2.68695	.881	-8.9419	4.9685
	White	-2.76737	1.13245	.072	-5.6987	.1640

* The mean difference is significant at the .05 level

Visual inspection of the ranges and standard deviations by ethnicity also showed some differences in the dispersion of the data, especially between Latinos (range=22; $SD=4.40$) and African Americans (range=20; $SD=5.18$), and Whites (range=16; $SD=3.74$), indicating that the data for White students were somewhat more tightly packed, while the data for the other two groups were somewhat more spread out. This implies that self-concept of ability for White students were somewhat more similar to each other than of Latinos or African Americans.

Not surprisingly, there were also differences in the means of self-concept of ability scores between levels of socioeconomic status. While visual inspection showed only small differences in the means prior to treatment ($M_{\text{Free lunch}}=20.0$; $M_{\text{Reduced lunch}}=21.4$; $M_{\text{No assistance}}=22.1$), a one-way ANOVA with pre-test scores as the independent variable and socioeconomic status as the dependent variable indicated a statistically significant difference at the .05 significance level in self-concept of ability prior to treatment between students who received free lunches and those who received no assistance. There was no significant difference between those who received reduced lunches and those in the other two groups (see Table 23).

Table 23. Comparison of Means of Self-Concept of Ability Prior to Treatment by Socioeconomic Status: Tukey HSD

(I) SES	(J) SES	Mean Difference (I - J)	Std. Error	Sig.	95% Confidence Interval	
					Upper Bound	Lower Bound
Free lunch	Reduced lunch	-1.53826	.87200	.184	-3.5957	.5191
	No assistance	-2.23052*	.65571	.002	-3.7776	-.6834
Reduced lunch	Free lunch	1.53826	.87200	.184	-.5191	3.5957
	No assistance	-.69226	.92862	.737	-2.8832	1.4987
No assistance	Free lunch	2.23052*	.65571	.002	.6834	3.7776
	Reduced lunch	.69226	.92862	.737	-1.4987	2.8832

* The mean difference is significant at the .05 level

Similar results were found in the analysis of data related to self-concept of ability prior to treatment between levels of prior knowledge. Visual inspection of the means indicated a larger difference between those who scored Far Below Basic ($M=20.7$) or Below ($M=20.3$) Basic on the California Standards Tests and those who scored Basic ($M=22.9$). A one-way ANOVA revealed a significant difference at the .05 significance level between the groups ($F(2,224)$, $p=.003$). The Tukey HSD test confirmed that these differences were statistically significant (see Table 24).

Table 24. Comparison of Means of Self-Concept of Ability Prior to Treatment by Prior Knowledge

(I) Prior Knowledge	(J) Prior Knowledge	Mean Difference (I - J)	Std. Error	Sig.	95% Confidence Interval	
					Upper Bound	Lower Bound
Far Below Basic	Below Basic	.69286	.65951	.546	-.8362	1.2489
		-2.14578*	.86508	.037	-4.1869	-.1047
Below Basic	Far Below Basic	-.69286	.65951	.546	-2.2489	.8632
		-2.83864*	.81952	.002	-4.7722	-.9051
Basic	Far Below Basic	2.14578*	.86508	.037	.1047	4.1869
		2.83864*	.81952	.002	.9051	4.7722

* The mean difference is significant at the .05 level

While the range of the data was slightly smaller for students who scored at the Basic level (18) compared to those at the Below Basic (22) or the Far Below Basic level (21), the standard deviations of the data for those Basic and Below Basic levels (4.18 and 4.13, respectively) were somewhat lower than that of the data for those who were at the Far Below Basic level (4.94), indicating slightly more similarity in dispersion for the two higher levels of prior knowledge.

Finally, visual inspection of the means of self-concept of ability scores prior to treatment between relative writing ability levels again showed little difference between lowest, low, and high ability writers and their self-concept of ability in mathematics prior to treatment ($M_{\text{lowest}}=20.9$; $M_{\text{low}}=20.8$; $M_{\text{high}}=20.5$; $M_{\text{highest}}=22.1$). A one-way ANOVA, however, found no significant difference at the $p<.05$ level in self-concept of ability prior to treatment between levels of writing ability ($F(3,217)$, $p=.474$).

In summary, analysis of the data related to self-concept of ability prior to treatment found many significant differences at the .05 significance level between various groups. There was a significant difference between males and females, with males having a higher self-concept of ability than females prior to treatment, White students scored significantly higher than Latino students, student who received no assistance scored significantly higher than those who received free lunches, and those at the Basic level of prior knowledge scored significantly higher than those at the Far Below Basic level. It seems, then, that boys, white students, those at the relatively highest socioeconomic status, and those at the relatively highest level of prior knowledge had higher self-concepts of ability prior to treatment than girls, Latinos, and those at the lowest levels of prior knowledge and socioeconomic status.

Writing ability was not significantly associated with self-concept of ability. The differences between groups suggest the need to control for the variables explored above in subsequent linear regression analyses.

Next, the data related to the differences in self-concept of ability between groups during treatment was analyzed. Visual analysis of the data indicates some differences between males and females during the treatment process. The range and standard deviation for males (range=2.25; $SD=0.47$) was not as great as the range for females (range=4.62; $SD=0.56$), indicating more variability in self-concept of ability for females than males during treatment. A comparison of means to 5% trimmed means and means to medians indicated little or no effect of extreme values on the data. While skewness values were within normal limits, the data for males was slightly negatively skewed (-0.187, indicating that a slight majority of their self-concept of ability scores fell above the mean. The skewness level for females, on the other hand, was slightly positively skewed (0.154), indicating that most of their self-concept of ability scores during treatment fell slightly below the mean for females. While the kurtosis level for females (3.93) was above the limit considered by many to be within normal distribution parameters, and the level for males (0.171) was within normal limits, the results of the Kolmogorov-Smirnov test of normality indicated that neither distribution was normal, indicating the need for non-parametric tests when comparing means and medians. Since visual inspection of the data showed the mean score of males (0.13) to be much higher than the mean score of females (0.09), the results of a two-tailed Mann-Whitney U test indicated that there was no statistically significant difference between the scores related to change in self-concept of ability during the treatment phase ($U=5394$, $p=.654$).

Visual analysis of the data showed a few differences between ethnicity in the dispersion of the data related to changes in self-concept of ability during treatment. Since there was only one Asian student, those data were not compared with the other groups. The range of the data for African Americans was somewhat higher than that for Latinos or Whites (range_{Latino}=2.38; range_{white}=2.00; range_{African American}=4.62), as was the standard deviation ($SD_{Latino}=0.46$; $SD_{white}=0.51$; $SD_{African American}=0.84$), perhaps indicating a greater diversity in self-concept of ability during treatment amongst African Americans than in the other groups. Skewness values were within normal distribution limits (-1 to 1) for all ethnicities, though all were slightly negatively skewed. However, the kurtosis level (4.53) for African Americans

was very high, indicating a high number of scores fell within a short range of data slightly above the mean. A comparison of the means ($M_{\text{Latino}}=0.09$; $M_{\text{white}}=0.11$; $M_{\text{African American}}=0.20$) again indicated a difference between African Americans and the other two ethnicities. Since the Kolmogorov-Smirnov test of normality indicated that the data for Whites was not normal and the kurtosis level for African Americans was not within normal distribution limits, a Kruskal-Wallis test was conducted to determine if the differences between the means was statistically significant. The differences in the change in self-concept of ability between different ethnicities was not significant ($H(2)=1.193$, $p=.551$). For the purposes of this comparison, the data from the Asian participant were not included in the data set.

While a comparison of means to 5% trimmed means indicated that no extreme values affected the data, a comparison of means to medians showed that there may have been some effect of outliers on the data relating to changes in self-concept of ability for African Americans ($M=0.20$; $Mdn=0.13$). However, all of the scores were found to be valid and left in the data set.

A comparison of the dispersion of the data related to changes in self-concept of ability and socioeconomic status revealed a difference in ranges between those who received free lunches (4.62) and those who received either reduced price lunches (2.37) or no assistance (2.13), while the standard deviations for both those who received free lunches (0.54) and those who received reduced price lunches (0.58) differed from those who received no assistance (0.46). Skewness levels fall within normal distribution limits for all socioeconomic groups. However, the shape of the data for those who received free lunches was highly peaked (5.266), again indicating some differences in self-concept of ability during treatment between those of the lowest socioeconomic level and those on the other levels. A visual comparison of the means again reveals differences between those who received free lunches ($M=0.08$) and those who did not ($M_{\text{reduced}}=0.12$; $M_{\text{noassist}}=0.15$); however, the results from a Kruskal-Wallis H test indicated that no statistically significant differences were found in the self-concept of ability scores between any of the socioeconomic levels defined in the study ($H(2)=1.163$, $p=.559$).

The dispersion of the data related to changes in self-concept of ability reveals some differences between students with a prior knowledge level of Far Below Basic (range=4.62;

$SD=0.57$) and those who scored Below Basic (range=2.38; $SD=0.49$) and Basic (range=2.38; $SD=0.52$). The skewness and kurtosis values for all levels of prior knowledge were within normal distribution limits, except for the kurtosis level in the data related to those with Far Below Basic prior knowledge (7.544). However, the Kolmogorov-Smirnov test of normality indicated that the data for those at both the Far Below Basic and the Basic level were not within normal distribution limits (Far Below Basic: $p=.003$; Basic: $p=.003$). A comparison of the means and medians for these same levels of prior knowledge imply that extreme values may have had an effect on the means (Far Below Basic: $M=0.06$, $Mdn=0.00$; Basic: $M=0.10$, $Mdn=0.00$), though a comparison of means to trimmed means does not necessarily support that finding. Since all scores were found to be valid, all data, including outliers and extremes, were left in the data set. The results of the Kruskal-Wallis test to compare the means between the groups indicated no significant differences between the means of the scores related to change in self-concept of ability nor between students with different levels of prior knowledge.

The greatest difference in the dispersion statistics for change in self-concept of ability during treatment between levels of writing ability was found between those with low writing ability (range=4.62, $SD=0.55$) and those with high writing ability (range=1.63, $SD=0.39$), the two middle levels. However, the greatest difference in standard deviation occurred between the high ($SD=0.39$) and the highest ($SD=0.71$) levels. Skewness and kurtosis values are within normal distribution limits for all levels except the kurtosis level for low writing ability (5.746). The Kolmogorov-Smirnov test for normality confirmed that the data for change in self-concept of ability for those with low writing ability did not fall within normal distribution limits ($p=.006$). A visual examination of the means also indicated a difference between the low writing ability scores and the scores for other levels of writing ability ($M_{lowest}=0.12$; $M_{low}=0.08$; $M_{high}=0.13$; $M_{highest}=0.11$). However, the results of the Kruskal-Wallis H test showed no statistically significant difference in the means of the scores related to change in self-concept of ability during treatment between levels of writing ability ($H(3)=1.018$, $p=.797$). Care must be taken, however, when drawing conclusions from these data since the numbers of subjects varied widely between levels ($M_{lowest}=36$; $M_{low}=108$; $M_{high}=66$; $M_{highest}=27$).

In summary, while differences in the dispersion and measures of central tendency appeared to indicate some differences between males and females, members of different ethnic groups, different levels of socioeconomic status, different levels of prior knowledge, and different levels writing ability, statistical comparisons of means within these demographic groups indicated no statistically significant differences in change in self-concept of ability during the treatment phase of the study between members within the demographic groups. As was the case with most of the data related to self-concept of ability, many of the data sets examined did not fall within normal distribution parameters.

The next section will examine the data related to the second research question of the study: Does the type of prompt affect self-concept of ability?

Research Questions #2: Does the Type of Prompt Affect Self-Concept of Ability?

The dispersion statistics related to the change in self-concept of ability by prompts types show some differences between the three data sets. The ranges in the data for the second and third, or procedural and self-monitoring (metacognitive), prompts were similar ($\text{range}_{\text{summary}}=2.63$; $\text{range}_{\text{procedural}}=3.50$; $\text{range}_{\text{metacognitive}}=3.25$), while the range for the summary prompt was the lowest of the three, implying a narrower spread of scores in the change of self-concept of ability after using the summary prompt. The standard deviation of the data for the third prompt was substantially higher than that of the other two prompts ($SD_{\text{summary}}=0.49$; $SD_{\text{procedural}}=0.50$; $SD_{\text{metacognitive}}=0.59$), indicating a wider spread of scores for the change of self-concept of ability after using the self-monitoring prompt.

The distribution of the data for the summary prompt was somewhat positively skewed (0.176), with most scores falling a little below the mean, while the distribution for the metacognitive prompt was very negatively skewed (-0.848), approaching the lower limit of a normal distribution and indicating that many of the scores lay above the mean. The distribution of scores following use of procedural prompts was extremely positively skewed (1.502) and leptokurtic, or highly peaked, (6.357), and well outside the parameters of a normal distribution. The distribution of the data related to scores following use of metacognitive prompts was also leptokurtic (2.155) and outside normal distribution parameters. Neither square root nor natural logarithmic transformations were able to correct for the extremes in skew and kurtosis for any of the data related to self-concept of ability. Consequently, all

inferential analyses, except for linear regressions, were conducted through non-parametric tests.

A comparison of the means and 5% trimmed means confirmed the effects of the outliers on the distribution of the data. While the mean and 5% trimmed mean of the data following summary writing were approximately the same (0.13), the mean following procedural writing is 0.09 and the 5% trimmed mean is 0.07. The mean following metacognitive writing (0.09) was much lower than the 5% trimmed mean (0.13), indicating the effects of outliers or extreme values on the statistics. Boxplots showed one outlier affecting the mean for summary writing, an extreme value affecting the mean for procedural, and several outliers and extremes affecting the mean for metacognitive writing. As above, all scores were retained in the data set, since all scores were found to be valid. The medians, unaffected by extreme values, were the same (0.0) following procedural and metacognitive writing, while the median following summary writing was 0.12.

In order to determine if there were statistically significant differences at the .05 significance level in the medians of the scores related to changes self-concept of ability between types of prompts, the Friedman Test was conducted. No significant differences were found in changes in self-concept of ability between types of prompt, $X^2(2)=.530, p>.05$.

In summary, while some differences were found in the data related to changes in self-concept of ability between types of prompts, no statistically significant difference at the $p<.05$ level was found. According to univariate analyses, then the type of prompt was not statistically associated with changes in self-concept of ability in this study. The next section will examine the effects of the type of prompt, as well as the other variables described in the study, on self-concept of ability.

MULTIPLE REGRESSION ANALYSIS FOR SELF-CONCEPT OF ABILITY

In order to explore the variables that affected changes in self-concept of ability (the dependent variable) in this study, four simultaneous regressions were conducted, one for each measure of achievement as independent variables. Other independent variables included pre-test achievement scores, self-concept of ability pre-test scores, prior knowledge, writing ability, and the demographic variables: ethnicity, gender, socioeconomic status, types of prompt, type of unit, and group. As with the regressions run for achievement, all output was

reviewed for problems with co-linearity by examining the Tolerance and Variance inflation factor (VIF). Variables with values less than 0.10 for Tolerance and over 30 for the VIF were removed from the analysis, as suggested by Pallant (2005). Correlations were also examined for values over 0.7 or below -0.7 and variables with these correlation coefficients were removed from analysis. Assumptions of normality were checked by examination of the Normal Probability Plot of Regression for each test run. No violations of this assumption were found. In addition, the scatterplot of the standardized residuals were all roughly rectangularly distributed, and a maximum of three outliers close to 3 or -3 was found for some tests. Given the magnitude of n for all tests, these outliers were not considered problematic. Finally, Cook's Distance was examined for each test run and no value over one was found.

Table 25 presents the most informative statistics contained in the models with the most explanatory power for each test run. Each of the models explains approximately 40% of the variance in self-concept of ability and is significant at the .05 level. In addition, each model contains of the same four independent variables and, in each model, the pre-test score for self-concept of ability makes the strongest unique contribution to explaining the change in self-concept of ability in this study ($\beta = -.623$ to $-.601$), followed by achievement pre-test scores, the change in achievement they made, and gender. For the achievement level measure, being in Group 2 also had an effect on achievement. Using the mean of the β values for purposes of illustration, the following four statements can be made about the independent variables in the first three models:

- Controlling for achievement change, gender, and achievement pre-test scores, every one point gain in self-concept of ability pre-test score there was associated with a -0.61 drop in the change of self-concept of ability.
- Controlling for achievement change, self-concept of ability pre-test score, and achievement pre-test scores, males were associated with a 0.23 point gain in the change of self-concept of ability.
- Controlling for achievement change, gender, and self-concept of ability pre-test score, every one point gain in achievement pre-test score there was associated with a 0.31 gain in the change of self-concept of ability.

Table 25. Simultaneous Regressions for Change in Self-Concept of Ability

Measure of Achievement	R ²	Sig.	Variables	Standardized Beta	Sig.
Percent Difference	.400	.000	Achievement change	.264	.000
			Gender (male)	.233	.001
			Pre-test score	.351	.000
			Self-concept of ability pre-unit score	-.623	.000
Achievement Change Ratio	.397	.000	Achievement change	.252	.000
			Gender (male)	.230	.001
			Pre-test score	.242	.000
			Self-concept of ability pre-unit score	-.615	.000
Percent Change	.382	.000	Achievement change	.195	.005
			Gender (male)	.228	.001
			Pre-test score	.347	.000
			Self-concept of ability pre-unit score	-.601	.000
Achievement Level	.384	.000	Achievement change	.175	.009
			Gender (male)	.222	.001
			Pre-test score	.262	.000
			Self-concept of ability pre-unit score	-.619	.000
			Group 2	-.147	.043

- Controlling for achievement pre-test score, gender, and self-concept of ability pre-test score, every one point gain in achievement change there was associated with a 0.24 gain in the change of self-concept of ability.

For achievement level change, the following four statements can be made about the independent variables:

- Controlling for achievement change, gender, being in Group 2, and achievement pre-test scores, every one point gain in self-concept of ability pre-test score there was associated with a -0.62 drop in the change of self-concept of ability.
- Controlling for achievement change, self-concept of ability pre-test score, being in Group 2, and achievement pre-test scores, males were associated with a 0.22 point gain in the change of self-concept of ability.
- Controlling for achievement change, gender, being in Group 2, and self-concept of ability pre-test score, every one point gain in achievement pre-test score there was associated with a 0.31 gain in the change of self-concept of ability.

- Controlling for achievement pre-test score, gender, and self-concept of ability pre-test score, every one point gain in achievement change there was associated with a 0.18 gain in the change of self-concept of ability.
- Controlling for achievement pre-test score, gender, achievement change, and self-concept of ability pre-test score, being in Group 2 was associated with a 0.15 decrease in the change of self-concept of ability.

While several variables in this study appear to have had an effect on self-concept of ability, the type of prompt did not.

The next section will analyze the data related to changes in achievement and changes in self-concept of ability in order to address the third research question in this study: Are self-concept of ability and achievement statistically related?

Research Question #3: Are Self-Concept of Ability and Achievement Related?

The relationship between achievement (as measured by pre-and post-test scores on unit tests) and self-concept of ability (as measured by scores on the self-concept of ability subscale of the Minnesota Mathematics Attitude Inventory) was investigated using the Spearman's Rank Order Correlation (ρ). Preliminary analyses were performed to determine if the assumptions of normality, linearity, and homoscedasticity were met. According to the Kolmogorov-Smirnov test of normality, the data related to three out of four measures of achievement, and the data related to self-concept of ability violated one or more of the assumptions above, requiring the use of the non-parametric Spearman's Rank Order Correlation (see Table 26). There was a small positive correlation between achievement gain measured as the Achievement Change Ratio and self-concept of ability ($r=.21$, $n=210$, $p<.01$), which meant that achievement by this measure helped to explain only about 4% of the variance of students' scores on the self-concept of ability subscale. There was also a small positive correlation between achievement as measured by change in achievement level and self-concept of ability ($r=.15$, $n=206$, $p<.05$), which meant that achievement by this measure helped to explain only about 2% of the variance of students' scores on the self-concept of ability subscale. While there were two statistically significant relationships found between achievement and self-concept of ability, the explanatory power of the relationships were quite small.

SUMMARY OF RESULTS

While factors varied from regression to regression, there was a clear association between prior knowledge, whether measured by the California Standards Tests or by pre-tests administered as part of this study, and achievement and change in self-concept of ability. Gender also seemed to be associated negatively with change in self-concept of ability, and also, perhaps with achievement change as well. Achievement change had a positive association with change in self-concept of ability and change in self-concept of ability had a positive association with achievement. Achievement change was positively associated with the type of unit, or curriculum. Finally, at least in some models, summary

Table 26. Correlation Matrix Between Achievement and Self-Concept of Ability

Spearman's Rho Correlations						
		Ach Percent Difference	Ach Change Ratio	Self-concept of ability Difference	Ach Percent Change	Ach Level Change
Achievement Percent Difference	Correlation Coefficient	1.000	.878**	.122	.545**	.819**
	Sig. (2-tailed)	.	.000	.078	.000	.000
	N	239	239	210	238	235
Achievement Change Ratio	Correlation Coefficient	.878**	1.000	.214**	.178**	.869**
	Sig. (2-tailed)	.000	.	.002	.006	.000
	N	239	239	210	238	235
Self- concept of ability Difference	Correlation Coefficient	.122	.214**	1.000	-.059	.148**
	Sig. (2-tailed)	.078	.002	.	.398	.033
	N	210	210	212	209	206
Ach Percent Change	Correlation Coefficient	.545**	.178**	-.059	1.000	.150*
	Sig. (2-tailed)	.000	.000	.398	.	.022
	N	238	238	209	238	234
Ach Level Change	Correlation Coefficient	.819*	.869**	.148*	.150*	1.000
	Sig. (2-tailed)	.000	.000	.033	.022	.
	N	235	235	206	234	235

** The mean difference is significant at the .01 level

* The mean difference is significant at the .05 level

writing appeared to have at least a small positive association with achievement change.

Quantitative analysis thus far suggests that only one type of writing students engaged in during the treatment period (summaries), as well as the type of unit, or curriculum, may have had a small effect on achievement, but not on self-concept of ability. The greatest effects on both achievement and attitude came from factors outside of the control of the educational environment of the study, especially prior knowledge, prior attitude, socioeconomic status, and gender. Serving as a bridge between quantitative and qualitative analyses, the next section will present mixed qualitative and descriptive quantitative results obtained from writing evaluations, teacher observations, individual interviews, and group interviews.

Evaluation Prompts: Group 1 Results

Participants in the study were asked to evaluate the effectiveness of the writing they did during each unit of study and to compare the effectiveness of the type of writing they did in one unit to that of previous units. Table 27 presents the frequency data related to Group 1, Unit 1 responses. Group 1 began with summary writing, then wrote about what they did and did not understand about the lesson, or self-monitoring (a type of metacognitive writing), and ended the experiment with procedural writing. After the first unit, 19 out of 26 students, or 73% of those who responded to the question without qualification and said that they believed that summary writing did help them learn the mathematics. Only seven students, or 27% of those who answered the question responded negatively, and one student did not respond to the question asked. Of those who said that summary writing was helpful, 20% stated that writing helped him remember the material, while only one student wrote that the summaries helped them understand the mathematics. Six students who gave positive responses said that their summaries served as good notes to help them remember later on and to study for tests (though one student who responded negatively said that he could not study from his summaries).

Table 27. Group 1 Unit 1 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	19	73%
Negative	7	27%
Not Addressed/Unclear Response	1	

In the second unit of study, this group responded to metacognitive prompts that asked them to write down what they did and did not understand about a lesson. These prompts were also called self-monitoring prompts. Participants were asked to respond to two evaluation questions. The first question was the same as that asked after the first unit of study: Did metacognitive writing help them learn, remember, or understand the math? The second question asked students to compare the two types of writing they had done so far by choosing the type of writing they felt was most helpful for them, and why.

Responses to the first question were similar to those after the first unit of study. Nineteen out of the 26, or 73% of the students who answered the question again indicated that metacognitive writing helped them learn the mathematics they were studying, though three of these students qualified their responses by writing that this type of writing was only sometimes helpful. Two of those who qualified their answers indicated that the mathematics they were studying at the time was sometimes difficult so the writing was not always as effective as when they understood the lessons better before writing. Five students who answered positively wrote that self-monitoring helped them remember the material, and one of these said that this type of writing helped them remember what they needed to work and what they could "...keep where it's at," making a specific connection to their own learning and the purpose of self-monitoring. One student found that self-monitoring helped him understand what he did and did not get from the lesson, while another said self-monitoring helped him realize what he knew about all the things we had done. Along the same lines, another participant who wrote that self-monitoring helped him a lot said, "I can think about what I know and understand the most and what I don't." Seven participants (23%) indicated that self-monitoring was not helpful, though two of these students reported that this type of writing did sometimes help them learn the math. Several students who responded negatively stated that they already knew the material or that all they really needed was to be shown how or to be given more problems. Five students referred to the writing they did as notes. Three of these students said that writing of this type was helpful as notes, but 1 student said he did not go back to the writing and the other said he was not able to go back and look at his notes, so the writing was not helpful.

When asked whether metacognitive or summary writing was more useful for them, eight participants (35% of those who responded to the question) chose metacognitive writing

as most helpful (one of these students chose summary writing, but described it as writing about what he did or did not understand), 12 (52%) chose summary writing, one wrote that both were helpful, two stated that neither were helpful, and four did not answer the question. Three students who chose metacognitive writing identified the personal nature of self-monitoring as their reasons for their choice. One said that he could see what they personally needed to work on and the others wrote that they could put down the “stuff” that was still hard for them. Another student stated that he could not write about what he understood in summary writing, the first type of writing he did, but could put in writing what he understood after self-monitoring. Of those who chose summary writing over metacognitive writing, three stated that summaries helped them remember the material better, while two others felt summaries were easier to write, and a sixth preferred the explanations provided in summary writing. One student said that summary writing “...gets my point across better,” while two other students chose summary writing by default stating that, “...writing down that you don’t understand doesn’t help,” and “metacognitive writing didn’t help at all.”

Tables 28 and 29 present the results of Group 1, Unit 2 responses.

Table 28. Group 1 Unit 2 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	16	61%
Qualified Positive	3	12%
Negative	7	27%
Not Addressed/Unclear Response	1	

Table 29. Group 1 Unit 2 Preference

Preference	Number	% (out of 23 respondents)
Summary over Metacognitive	12	52%
Metacognitive over Summary	8	35%
Both	1	4%
Neither	2	9%
Not Answered/Unclear	4	17%

The final evaluation again consisted of two questions: (1) Whether or not the type of writing they had been doing in the third unit of study helped them learn the mathematics; and

(2) Which type of writing (summary, metacognitive, or procedural) was most helpful to them.

In the final evaluation, 20 students (74%) stated unequivocally that procedural writing helped them learn the mathematics in some way, two (7%) responded positively with some qualifications, four (15%) wrote that procedural writing was not helpful, and one student was not sure. Six of those who responded positively, with or without qualifications, indicated that procedural writing helped them remember the material better, five said this type of writing helped them understand the problem better, and five thought procedural writing provided good notes. Of those who liked procedural writing as notes one student specifically mentioned the advantage of reading notes in his own words. Of the four students who did not think procedural writing was helpful, two indicated that they did not need writing to learn the math while one student expressed his inability to write about things he did not understand or learn while one just didn't like writing, and the other stated that he did not notice if procedural writing helped or not. The same student, however, chose procedural writing as the most helpful of the three types on the second question of the final evaluation.

In response to the second question, which type of writing helped them most, 12 out of the 24 students (50%) who answered the question chose procedural writing, five (21%) chose metacognitive writing, four (17%) selected summary writing, one chose both procedural and metacognitive, and one wrote that none helped. Of those who chose procedural writing, four mentioned the step-by-step nature of procedural writing as helpful but did not, or were unable to state more specifically why the steps helped them. One student did write that the steps showed how to start and end a problem the right way. Another student who chose procedural writing said that it, "Explains what you do each step you do," possibly referring to the added depth of procedural writing with explanations or examples of each step, and a second student also indicated that procedural writing explains more. In addition, two students stated that procedural writing was easier for them and three students indicated that procedural writing helped him remember how to do a problem. Of those who chose summary writing as the most helpful type of writing, two indicated that their summaries were used as notes and another may have confused this type of writing with procedural. Of those who chose metacognitive writing as the most helpful of the three, one wrote that it helped them remember the material, and another referred to the feedback he thought it would provide the

teacher. Two students referred to the intended purpose of self-monitoring. One student said that it helped him know what to work and another said that it helped him understand what he didn't know. Writing as notes was given as a reason for choosing both summary and procedural writing.

Tables 30 and 31 present the results of Group 1, Unit 3 responses.

Table 30. Group 1 Unit 3 Responses

Response	Number	% (out of 27 respondents)
Unqualified Positive	20	74%
Qualified Positive	2	7%
Neutral	1	4%
Negative	4	15%

Table 31. Group 1 Unit 3 Preference

Preference	Number	% (out of 23 respondents)
Summary	4	17%
Procedural	12	52%
Metacognitive	5	22%
More than one	1	4%
None helped	1	4%
No response/unclear	4	

Evaluation Prompts: Group 2 Results

Group 2 began with procedural writing, then wrote summaries, and ended with self-monitoring, or metacognitive writing. Table 32 presents the results of Group 2, Unit 1 responses. After the first unit, approximately 69% of those who began with procedural writing stated unequivocally that procedural writing was helpful for them. Nineteen percent of the group stated that writing did help them somewhat, only one student was neutral (4%), and two students (8%) did not directly answer the question. In Group 2, 43% of those who stated that procedural writing was effective for them said that procedural writing helped them understand the content better. One student specifically credited the process of revision for helping them figure out the problems at times. In addition, 26% of the positive respondents indicated that writing helped them remember the content, while 17% stated that procedural

writing helped them develop a visual representation of the problem. Finally, 30% of these students felt the writing they did could be used as notes for tests and other assignments.

After Unit 2, 77% of the students gave unqualified positive responses to summary writing. Of those, three wrote that summary writing helped them understand the content, while eight said that summary writing helped them remember the math, and seven said summaries were helpful as notes to refer to before tests. Two students indicated that summaries also helped them to know how to solve a problem, providing them with procedural knowledge. One student said that writing summaries helped him express how he saw the math through his own eyes, and the other said summaries would help him in the future. Of the two students that gave qualified positive responses, one said it helped him remember the math; the other said the summaries were helpful as notes. Of those who wrote that summaries were not helpful, one indicated that we never looked at them, implying that they were useless as notes (though others did look at them for notes), and one said writing summaries got him “messed up,” or confused.

Table 32. Group 2 Unit 1 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	18	69%
Qualified Positive	5	19%
Neutral	1	4%
Not Addressed/Unclear Response	2	8%

When students were asked to compare summary writing to procedural writing, Post-Unit 2 results for Group 2 revealed that most students (70%) preferred procedural to summary writing. Students gave several reasons for preferring procedural over summary writing. Three students wrote that procedural writing helped them understand better than summary writing, two said that procedural writing helped them remember better than summaries, three indicated that procedural writing was easier than summary writing, and one student wrote that procedural writing could be used for notes better than summaries. Other participants wrote that procedural writing helped them “work better” than summaries and that procedures were more fun than summaries. Two students mentioned that procedural writing was easier for them to understand than summary writing. Four students stated that the step by step nature of procedural writing was better for them but gave no reason why.

On the other hand, 43% of those who thought summaries were better for them than procedures said that summary writing helped them understand the content better and two students indicated that summary writing contained more information, both procedural and understanding, than procedural writing alone. Another student stated that summary writing made him write down what the examples means and two participants indicated that summary writing was easier than procedural writing for them.

The data for Group 2, Unit 2 responses are presented in Tables 33 and 34.

Table 33. Group 2 Unit 2 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	20	77%
Qualified Positive	2	8%
Negative	2	8%
Not Addressed/Unclear Response	2	8%

Table 34. Group 2 Unit 2 Preference

Preference	Number	% (out of 24 respondents)
Procedural over Summary	17	70%
Summary over Procedural	7	30%

After Unit 3, only 54% of the group gave unqualified positive responses to metacognitive writing, while 15% gave qualified positive responses, and 23% wrote that metacognitive writing did not help them. Out of the students who responded positively, both qualified and unqualified, to the question about whether or not metacognitive writing helped them learn the content, two students mentioned that metacognitive writing helped them because they were able to express their questions or what they did and did not understand but only one student was able to link the expression of their questions to remembering the content. Five other students said that writing about what they knew and didn't know helped them remember how to do the problems, but one student stated that it did not help him with subsequent problems of the same type. Three out of the students who responded positively to metacognitive writing related it to increased understanding and four continued to associate writing with notes for future reference. However, one student who responded negatively to this type of writing said specifically that he did not think it was effective because he could not use it for tests. Some students seemed to find it difficult to evaluate specifically how it

helped them. One student wrote that it helped them just to know what he did not yet know; another wrote that just explaining what he knew was helpful, and another implied a more personal aspect, more ownership of what he wrote, and said that the writing was his, with no one showing him. Three students who responded positively to metacognitive writing identified feedback to the teacher as a benefit of this type of writing, assuming the teacher would read it and respond to their questions. One student who responded negatively, however, stated that his/her problem with metacognitive writing was that the teacher would not answer questions at the time of writing, only ask the student to write down the question. Other reasons for negative responses were that the student already understood everything, that the writing confused them, and that they wanted more math problems; not writing about math problems.

When asked to choose the type of writing that helped them most, half of the participants in this group chose procedural writing over summaries and metacognitive writing, while 15% chose summary writing, and 12% chose metacognitive writing. One student could not choose between procedural and summary writing as the best for him because he liked both step-by-step writing and noting important information, and one student said that all three helped him in some way. Only one student wrote that none of the writing was helpful, one was unclear in his response between procedural and self-monitoring, and six participants in this group did not respond to the question. Of those who chose procedural writing as the most beneficial, one wrote that it helped him with his skills, three said procedural writing made good notes, one said procedural writing was easiest and helped him remember how to do the problems, and one wrote that procedural writing helped him understand the material because it, "...gets it in my head faster." One student who chose procedural writing over the others stated that summary writing did not help because it only addressed the important things, not the how to, and that he sometimes did not "get" the self-monitoring at times. Of those who chose summary writing as the best type of writing for them, one said it gave him all he needed to know, another wrote that it helped him to have to explain the way he did in summary writing, and another wrote that it helped him with new types of problems. Only one mentioned notes as the reason for choosing summary writing. Of those who chose self-monitoring as the best type of writing for them, one said it was easiest, another said it helped him visualize the content (though it got annoying after awhile),

and another stated that it helped him understand because it helped him know what to ask the teacher. The one student who said that all types of writing were helpful to him wrote that the explanations common to all types were what made writing helpful to him. Several students who did not choose one type of writing over the other gave benefits and/or drawbacks to each type of writing or to writing in general. One student said that writing only helped when he already understood what was being taught while another said the blue book was a guide on how to do the problems we had addressed. Finally, one student ran down the list, giving their assessment of each type of writing. He said that in procedural writing it was hard to write all the steps, summary writing was easier, and metacognitive writing was the hardest because "...you have to think."

The data for Group 2, Unit 3 are presented in Tables 35 and 36.

Table 35. Group 2 Unit 3 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	14	54%
Qualified Positive	4	15%
Negative	6	23%
Not Addressed/Unclear Response	2	8%

Table 36. Group 2 Unit 3 Preference

Preference	Number	% (out of 19 respondents)
Summary	4	21%
Procedural	9	47%
Metacognitive	3	16%
More than one	2	11%
None helped	1	5%
No response/unclear	7	

Evaluation Prompts: Group 3 Results

After the first unit of study in which students engaged in self-monitoring, 23 students in Group 3 responded to the evaluation survey. The results of this survey are found in Table 37. Of these, 18 (78%) indicated that self-monitoring helped them learn the math they were studying with or without qualification and 5 (22%) gave a negative response to the question. Only one student who gave a positive response said that self-monitoring helped him

remember the material while six students who gave a positive response wrote that metacognitive writing of this type helped them understand the mathematics, only one positive respondent mentioned feedback as the reason he found this type of writing helpful, and three indicated that they used their writing as notes. Of those who said self-monitoring helped them with understanding, one also indicated that he used his writing as notes, as a review and source for asking questions in class. One student indicated that writing in his own words, facilitated by self-monitoring, helped him understand the material, “much better,” and another said that self-monitoring kind of helped because if he wrote in his own words, no one is showing them how, implying independence. Three students gave insight into how self-monitoring helped their thinking. One stated, “I keep thinking about what we’re learning and that way I do better.” The second student said, “It makes me think better and a lot harder,” and the third said it helped him explain his answer. The reasons for negative responses are similar to those given by the other groups for other types of writing, especially that these students prefer working on problems, not writing about them, and they already know what they know. One student also wrote that the words were too long and that there was too much information, possibly indicating his dislike for writing in general compared to the relative shorthand of calculation alone.

Table 37. Group 3 Unit 1 Responses

Response	Number	% (out of 23 respondents)
Unqualified Positive	18	78%
Negative	5	22%

During the second unit of study, this group responded to procedural prompts. In response to the first evaluation question after this unit, 27 students wrote whether they thought procedural writing helped them learn the mathematics they were studying. Twenty-three of the students (85%) wrote that procedural writing did help them, at least some of the time. Four students (15%) wrote that procedural writing did not help them, one indicating that he already knew how to solve the problems, one said it was a waste of time but did not say why, one said it was too hard, and the last said he was just not used to it. Of those who said procedural writing was helpful in some way, only four said that this type of writing helped them remember the material, while six said it helped them understand the lesson better. One of those that wrote that procedural writing helped him understand the math

also said that the writing made him think about the steps. Another student who gave a positive response also indicated that the breaking down of the problem into steps was somehow helpful to him, as was also noted in the Group 1 results. One student who wrote that procedural writing helped him remember credited the repetition of writing for getting the material to stay in his mind, while another student stated that procedural writing improved the way he saw the problems.

When asked to compare metacognitive writing, or self-monitoring, to procedural writing, 11 out of the 19 participants (58%) who answered the question chose procedural writing over self-monitoring. Four of these students wrote that procedural writing helped them understand better, two said it helped them remember better, one (not the same student as in the previous paragraph) wrote that it helped him see the problems better, one indicated the usefulness of procedures as notes, and one said it explained the problem better for him. Several of these students again referred to the step-by-step nature of procedural writing, or the breaking down of problems, as helping them learn. One student referred to the social construction of knowledge that took place while writing procedures as a help to their learning, a phenomenon that would not take place while writing about what an individual did and did not understand about a problem. Of the five students who chose self-monitoring over procedural writing, two stated that it helped them identify and/or focus on what they did not understand. Two students indicated that both types of writing helped them learn. None of these students wrote that neither type of writing was useful for them.

Tables 38 and 39 present the results of Group 3, Unit 2 responses.

Table 38. Group 3 Unit 2 Responses

Response	Number	% (out of 27 respondents)
Unqualified Positive	17	63%
Qualified Positive	6	22%
Negative	4	

Table 39. Group 3 Unit 2 Preference

Preference	Number	% (out of 19 respondents)
Procedural over Summary	11	58%
Summary over Procedural	6	32%
Both	2	11%

After the third unit in which these students wrote summaries, only nine of the 27 students (33%) who responded stated that summary writing was helpful to them, while 17 students (63%) indicated that summary writing was not helpful. Three of those who did not find summaries helpful said that they were not able to go back and use them as notes, indicating, perhaps, a preconceived idea about how summaries are used. Five students wrote that summaries were complicated or confusing, with one student preferring the step-by-step clarity or simplicity or procedural writing, and seven participants characterized this type of writing as a waste of time or did not even know what they were writing about. One student inferred insightfully that metacognitive writing was more comprehensive in some ways because summary writing was only about what you know but in metacognitive writing, "...you get to ask about what you don't understand."

Of those who wrote that summaries were helpful to them, five students said that summaries helped them remember the material, and one specifically wrote that summaries helped him especially remember the important parts of a lesson, not all of it. Only two students indicated that summary writing helped them understand the material, though one of those was very positive about summaries, saying they not only helped him remember and understand quickly, but also was easier to understand, was fun, and good for review. Another student also wrote that summaries were good for notes. Only two students who responded positively wrote that summaries helped explain the material for them, and another said it was an easier kind of writing.

The results of Group 3, Unit 3 responses are found in Tables 40 and 41.

Table 40. Group 3 Unit 3 Responses

Response	Number	% (out of 26 respondents)
Unqualified Positive	9	35%
Negative	17	65%
Not Addressed/Unclear Response	1	

Table 41. Group 3 Unit 3 Preference

Preference	Number	% (out of 27 respondents)
Summary	3	11%
Procedural	14	52%
Metacognitive	7	26%
More than one	1	4%
None helped	2	7%

When asked to choose which of the three types of writing was most helpful for them, 14 out of the 27 students (48%) who responded to the question preferred procedural writing over the other two types, three (11%) chose summary writing, seven (26%) preferred self-monitoring, two stated that none of the writing helped them, and for one student, all three types were helpful. One of those who preferred procedural writing stipulated that the procedure had to include why each step was taken to be helpful and also made a connection between mathematics and literacy, saying that, "Writing and reading are big concepts in math, they all squeeze together." Two other students who chose procedural writing also indicated that metacognitive writing was helpful, too. One of these students wrote that procedural writing helped him with metacognitive writing, connecting writing to his growth in understanding. The other student stated that he preferred asking questions after the teacher explains the skill or concept. Another student who chose procedural writing also indicated that summary writing was helpful, too, though he did not elaborate. Two students who preferred procedural writing mentioned it was good for notes and one said it showed him how before they actually practiced solving more problems. Only two students who chose procedural writing over the other types said it helped them remember the material, while one of those who chose summary writing specifically said that summarizing was easier and helped them remember the important information and that he would forget the material if he tried to remember it all. Another student who chose summary writing also said it was easier for him. One student chose procedural writing by default, saying that he sometimes did not understand the metacognitive writing and that summary writing did not help since it only emphasized the important parts of the lesson. Those who chose metacognitive writing did so because it helped them express what they did or did not know, helped them by asking questions, or helped them understand what they did or did not understand. One of the students who wrote that none of the types of writing was helpful said he sometimes did not understand his own writing and that he was never given the time to use the writing for review, and the other simply said that he was good at math, implying no need for writing. The student who wrote that all types of writing were helpful did not elaborate.

By the end of the study, 39 out of the 71 students (55%) who responded to the question asking them to choose the types of writing that helped them most chose procedural writing. Only 20% (14 out of 71) chose metacognitive writing in the form of self-monitoring

and 15%, or 11 out of 71, chose summary writing as the most helpful type of writing for them. Five students (7%) said that none of the types of writing were helpful, and two students (3%) indicated that all three types, procedural, summary, and metacognitive writing helped them learn mathematics.

In summary, even though summary writing was the type of writing in this study that had even a small effect on achievement, the results of writing evaluations indicate that most students did not prefer this type of writing over the others learned over the course of the treatment periods. Students had many reasons for their writing preference. Some of which were stated above. The next section will present qualitative data, much of it in their own words, from not only writing evaluations, but also individual and group interviews conducted several months after the treatment period that describe students' thoughts and feelings about the types of writing in which they engaged. For the remainder of this paper, students who participated in the interview process will be referred to by the pseudonyms they chose during that process. Those who did not participate in the interviews will be referred to by the numbers used to process their quantitative data.

QUALITATIVE RESULTS

Procedural Writing

As the results above suggest, procedural writing was favored by a majority of the students for a variety of reasons, some of which are easy to generalize, and others that are unique to individual students. Procedural writing included not only writing the steps to a problem solution, but also supporting those steps with some type of elaboration, most often references to an example, preferably made up by the student. One of the most often mentioned reasons for preferring procedural writing over summaries and self-monitoring was simply the step-by-step nature of procedural writing. While many students simply referred to the step-by-step nature of procedural writing as being helpful to them (#16, #36, #41, #48, #6, #9, #12), some students made connections between one step and the next on problems. Student #51 chose procedural writing over self-monitoring for just this reason. Student #53, who seldom participated in the writing process without much encouragement, said that she liked procedural writing because she could understand it and it helped her remember "...how it goes by steps." Procedural writing seemed to be faster, easier and/or easier to understand

for several students (#12, #13, #16, #22). For many student throughout the study (#6, #8, #12, #15, #19, #21, #28, #38, #52, #53, #54, #59, #62, #66, #75, and #77), procedural writing helped them remember how to do problems on tests and on future work.

In addition, several students referred to the process as “breaking it [the mathematics] down” (#42). Miranda said in her second evaluation that procedural writing was better for her than self-monitoring because by “breaking it down” for her, she understood the math better. Miranda repeated this process of “breaking it down” as important to her because she did not understand if the problems were not explained step by step. She even mentioned that self-monitoring and using examples were important in that they helped her identify steps she did not understand and to get help on those steps. For Miranda, as for many others who struggle in mathematics, mathematical problems seem to be series of steps that must be mastered, not whole problems that can be solved in a variety of ways. Student #15 preferred procedural writing because it taught her “...what's next and how to do it all the way through from step one to the last step.” Student #23 chose procedural writing over summary writing on his second evaluation because the steps showed him what to do first, perhaps implying that once he knew the first step, he would remember the other steps, and student #44 chose procedural writing over self-monitoring because he recognized that procedural writing helped him both learn how to do the problem and see if he knew how to do it at the same time, incorporating self-monitoring into the process as well.

Metacognitive Writing (Self-Monitoring)

Self-monitoring was the second most popular type of writing in this study. In self-monitoring, students were asked to write what they did and did not understand about a lesson, with examples of each included in the text. While several students indicated that self-monitoring was helpful for them, only a few recognized and clearly expressed the true purpose of self-monitoring. Most students who indicated self-monitoring was of use to them simply restated that it showed them what they did and did not understand, the necessary components of self-monitoring given by the teacher in her directions and examples to the class. Some students simply appreciated the opportunity to ask questions or express their feelings about their learning (#43, #45, #38, #17). Student #1 went a little farther when she said that if she asked questions to herself, it usually helped her “...express every question

I've had about math and I can even write if I understood it or not." Student #32 chose self-monitoring just because she could ask questions, and student #51 felt self-monitoring was the easiest type of writing to understand. Others, like student #34, felt self-monitoring was best because the other types of writing were more confusing to them. The same student also wrote that self-monitoring helped her understand what she learned.

Some, however, showed insight into the higher-order thinking inherent in metacognitive writing. In her second evaluation, Wendy said she liked the self-monitoring better than procedural writing because she could tell whether or not she knew something. By the time of the interview, she clarified that when, through the writing process, she found that she did not know the content, she would then ask her classmates or the teacher for help. Student #38, a Latina with below basic prior knowledge and lowest level writing ability, showed such insight on her second evaluation when she chose self-monitoring over procedural writing because, "...I actually asked myself if I don't get it then I would focus on it." Student #6 wrote that by self-monitoring she knew what she did not know..." Student #19 wrote that self-monitoring helped him understand what he was learning and how to answer his problems, while student #34, who felt self-monitoring was best because the other types of writing were more confusing to them, wrote that self-monitoring helped her understand what she learned. Student #24 said that self-monitoring helped him remember the "stuff" he needed to learn because he was asking questions and writing them in his blue book. On the other hand, student #26 said he did not like self-monitoring because, "...you have to think." He thought summary writing was easier because all he had to write about was what he had done.

Summary Writing

As mentioned above, summary writing was the least popular yet, statistically, the most effective of the writing types in this study. In summary writing, the students were asked to identify and write about the important points in a lesson. An English teacher from the same school site as the teacher/researcher suggested giving students a set number of points on which to focus and recommended they try to find three points about which to write. In the group interview for Group 1, the interviewer asked students if they had done summary writing in any of their other classes at school. They affirmed that they had written summaries

in English and sometimes in history. However, even though summary writing was probably the most familiar to them, the participants in the Group 2 interview said that the summaries they wrote in English were different from those they were asked to write in math. They said that the writing in English did not have to be “perfect” like it did in math, perhaps referring to the difference between informational writing and more expressive writing. Writing in classes other than mathematics was also mentioned in the Group 1 interview. The participants there again said that procedural writing in English was not the same as writing in mathematics, with one student simply explaining that the writing was for two different subjects, so there could be no similarities or interaction between the two. These students also did not see any relationship between the reading response journals, a type of summary, and the summary writing they had done in math. However, in the Group 3 interview, one student said that he had done some procedural writing in another class then wrote about how he understood the lesson, a combination of procedural writing and self-monitoring that he enjoyed and clearly recognized that the two types of writing were related. Unfortunately, the participant did not mention the class in which this writing took place.

Few students chose summary writing as the most helpful and those who did sometimes had a difficult time expressing how it helped them. Student #32 simply said that explaining was helpful when she did not know the content. Student #7 said she thought summary writing helped her because “...when I write a summary and give an example it makes me think how to do it and understand.” She went on to say in later evaluations that since she had to use explanations in her summary writing, she had to learn the problem. For others, like student #14, summaries were easier for them and helped them understand more of the content, while for Taylor, summary writing “...helps me express how to see math in my eyes.” When asked in his interview how he liked the writing in math, Taylor generalized his appreciation for the opportunity for self-expression to all three types of writing: “...it was good because it gave, like, all the students a way to express themselves in, like, their own words how—what they can improve on or how they felt about [an] activity.”

Other students who felt summary writing was helpful were more specific about its benefits. Student #42 recognized the specific benefit of summary writing when he chose it over both procedural writing and self-monitoring, stating that summaries helped him remember only the important things about a skill or concept, since he sometimes forgot

everything if he tried to remember everything. Student #10 felt that summaries had more information and “said more things” than procedural writing, an observation shared by student #11. Omar felt that summary writing was helpful because it helped him remember the main points, a viewpoint shared by student #21, who said that summary writing helped her remember what was important, helped her understand the mathematics, and was easier to do than procedural writing. Students #23 and #26 also mentioned that summary writing helped them remember the main or important parts of what they had learned. Juke, who said in his interview that all three types of writing were “fun” and wished the class had continued to use the blue books after the conclusion of the study, was very specific about summary writing on his second evaluation, stating that it helped him remember the important points on a test. However, for Juke, procedural writing was more fun than summary writing.

Themes

Several qualitative themes emerged throughout the course of this study, including resistance, the inability to write when the content is not fully understood, the inability to clearly identify and communicate students’ feelings about mathematics, elaboration, the effects on curriculum as a result of reading student work, writing as a reference, getting the math “stuck” in their heads, and grades. Most, such as resistance, elaboration, the effects on curriculum as a result of reading student work, and grades, were anticipated from past experience and study of the literature surrounding writing in mathematics. Others, such as the inability to write when they do not understand the content and the inability of students to identify and clearly express their thoughts and feelings about writing were not anticipated from the literature.

Resistance

The first theme that emerged was that of resistance. Resistance came in several forms, some anticipated and others as a surprise to the researcher. Some of the resistance encountered was strong and overt, some was passive and very subtle. From the beginning of the data collection stage, at least some students resisted writing in their mathematics class, no matter which type of writing they were using. Teacher observations, student evaluations, and interview data all support the theme of resistance. When students were first introduced to writing in math during the study, groans could be heard around the classroom. A few

students stated that they were in math class, not English, an opinion also expressed in the first evaluation of self-monitoring by student #47. As the study progressed, students became more comfortable with writing. However, several students continued to resist in a variety of ways. Some would simply write down the prompt and not respond, others would not write anything, even after several attempts to positively encourage students. Still others would write, but only to meet the criteria of the assignment, not to really reflect on what they learned. Five categories of resistance are identified here:

- General resistance to both writing and mathematics;
- Resistance to writing in general;
- Resistance to specific types of writing;
- Resistance to elaboration
- Resistance to writing when it took the place of working math problems.

As mentioned above, several students would not write, even for the points awarded for completion. These students generally received from one to three points on their writing assignments (0- not attempted; 1- problem written, no further response; 2 or 3—some response; 4—adequate response but may not have included elaboration; 5 complete response, including example or other type of elaboration), except when the class was given additional support due to general discomfort with the content. This additional support generally came in the form of extended notes on the board from which students could draw in their writing.

One of these resistant students (#9), a below basic Latino with average writing ability and no lunch assistance, did very little writing but was generally positive on his evaluations. Since he averaged only 53% on his work grades in the class (work grades included both class work and homework and was graded mostly by completion, not correctness), his resistance was probably not to just the writing, but to mathematics and schooling in general. Another of these students (#45) was, again, a far below basic Latino with free lunch and low writing ability. On his first evaluation of self-monitoring, he indicated that the writing helped remove some of the confusion he had been experiencing in mathematics, and on his second evaluation he indicated that procedural writing helped him understand more. His response to the third evaluation of summary writing was negative, but he mentioned that it did let him express himself.

Several other students fit into this category, and though most gave positive responses to evaluations in general, they wrote little and did little work in the class. One (#36), a far below basic Latina with free lunch and the lowest writing ability, was very resistant to writing, especially at the beginning of the study. On her first evaluation of self-monitoring, she said that she felt the writing was not helpful because there was too much information and the words were too long to remember. On her second evaluation, she again said that the writing was too hard, but that procedural writing helped her more. On her last evaluation, she said that summary writing helped because it explained what the class was doing and that all types of writing were helpful to her. However, her blue book entries continued to be incomplete except when much support was provided to the whole class.

Probably the student with most total resistance (#74) was a below basic African American male with no lunch assistance and the lowest writing ability. His responses to evaluations were succinct and generally negative. His first response to the question whether summary writing had helped in any way was, simply, "No." On his second evaluation of self-monitoring, he said that the writing did not help him because once he was shown how to do something, he automatically "gets it." He also said that summary writing was better because self-monitoring did not help him at all. On the final evaluation, his responses were negative for any type of writing. This student was also resistant to more than writing. His average work grade was only 53%, and he often refused, both passively and actively, to complete his work.

Most students in the resistance category were only partially against writing, for a variety of reasons, and were more motivated in general to do well in school. They also recognized the value of writing at least some of the time. A student in this qualified resistance category was a below basic African American male who chose to be a part of the interview process and called himself Most-Wanted (from a video game). Most Wanted, also had average writing ability and no lunch assistance. This student, however, was more motivated by grades in general, earning a 70% average in his work over the semester. Throughout the study, it was very difficult to get this student to write. Both teachers in the classroom had to both encourage and warn about consequences in order to get him to write. In his first evaluation on summary writing, Most Wanted started off with a negative response. He said that he already knew how to do the mathematics and that it was easy for him. On

subsequent evaluations and in his interview, his responses were more positive, especially toward self-monitoring because the writing helped him remember more. However, during his interview, Most Wanted also stated that he did not like writing in general, not just in math, even though he felt it did help him.

Another student who showed resistance to writing while recognizing its value under some circumstances was a below basic Latino with no lunch assistance and high writing ability who called himself Cristiano during the interview process. Cristiano earned an average of about 72% on his work grade in the class, but his grades varied widely from 47% to 100%. He did very well on tests, always above 80%. From the outset, Cristiano expressed his resistance to writing. In his first evaluation of self-monitoring, he responded that writing did not help him at all. On his second evaluation on self-monitoring, he responded that it usually did not help him because he already knew the material. This same opinion was echoed by other students for this and other types of writing (#14, #35, #43, #44, #81). In his blue book, Cristiano would never write what he did not understand, though he also refused to elaborate on what he did know. On the third evaluation, he did say that procedural writing was best because it explained more and it was “kinda good” because he could look back at it to remember the material, though in his interview, he implied that he just said he liked procedural writing to placate the teacher. Cristiano also reiterated in the interview that he did not like the writing because it did not help him because he already knew the content. However, he also said that writing was helpful at the beginning of learning, but that when you “keep on doing it, it just gets, like, annoying,” and later in the interview said that it was boring. Cristiano also said that writing was helpful when he did not get a problem write. If he got help and then wrote it down the right way, he felt he could remember the steps better. An important possible difference between Cristiano and many of his peers was that he could get a lot of help from his parents. He mentioned in his interview that they would help him not only with the basic material, but also with possible permutations of the same type of problem (i.e. When solving equations, the variable may appear on either the right or left of the equal sign, the variable term may be added or subtracted from a constant, etc. Each situation adds a new twist to the solution process). Cristiano also mentioned that the inclusion of examples in his writing helped him remember how to do the problem later on during a test.

Student #35, who called herself Miranda for the interview process, a far below basic African American female with low writing ability and free lunch, was also resistant to writing but was motivated to do well in school. Her average work grade was 78%, relatively high in this sample of students. Though Miranda did much of the writing required in the class, she had to be encouraged often to write more in response to the prompts and include elaboration in her writing. In her evaluations, she thought procedural writing helped her but that the other types of writing did not, stating that she did not like reading or writing in any class, that writing in math was a “waste of time,” and, “If I need help with a problem, writing does not help.” In her interview, though, Miranda indicated that she did enjoy the writing and that it helped her understand what to do. She even stated that procedural writing helped her raise her grade. The interviewer asked Miranda if she had changed her mind, and she said she had changed her mind because it kind of helped, but only if the problem was “broken down” so that she could understand it. Even though she often wondered why she was writing in math and why the teacher was making her write, she did come to see the benefits of writing by the time the interviews took place several months later.

Several students resisted only certain types of writing, especially self-monitoring. Many students were unwilling to write what they did not understand or to ask questions in writing, even when instructed to do so (#7, #16, #44, #46, #81). Students seemed unable to write the same questions they were asking the teacher aloud, expecting instead an immediate answer to their questions, which they would then explain in their writing as if they understood the concept or skill prior to responding to the prompt. Their resistance did not result in a lack of writing, but in some components of the writing. One student (#16) said in their evaluation of self-monitoring that the blue books, where they did their writing, was for things they understood. This student never did comprehend the purpose of self-monitoring. Another student (#44) stated that when he wrote the things he knew how to do, that meant he remembered them. But when he did not understand something, it was not the writing that helped, it was asking how to do it. He did not get the idea that writing what he did and not understand would clarify those boundaries in his mind. Student #46 wrote that she already knew what she did and did not know. She did not see the purpose in putting any of it in writing. Even though his evaluations of writing were generally positive, student #7 stated in

his evaluation of self-monitoring, “What helps me is working on what I need help on not writing it down.”

A fourth form of resistance was resistance to elaboration. The teachers often had to remind students to include examples in their writing or say why a step was important in a procedure or a term or concept was important in a summary. Even in self-monitoring, students were asked to communicate more effectively by not only explaining what they did understand, but by showing at what point in a problem they need help. Since the participants had a very difficult time with elaboration in general, the teacher decided to focus on the inclusion of examples with only an occasional exhortation to other forms of elaboration. A good example of this kind of resistance is Miranda, student #35 mentioned above. Miranda mentioned in her interview that that she did not like the constant reminders she got to include examples when writing. But Miranda was not alone. Only about 15% of the students in the study consistently used examples or other ways to elaborate their responses, despite frequent reminders and very clear examples for each type of writing.

Many students who did much of the writing indicated their resistance to writing in math in place of actually working on math problems through their evaluations and interviews. Writing to them had no place in mathematics and they made little or no connection between language and mathematics. Student #47, a below basic African American male of low writing ability and who received free lunch, stated in his first evaluation of self-monitoring that writing did not really help because “all you’re doing is writing”, and called the writing a “waste of time” in his last two evaluations. Student #43, who called himself Piggy for the interview process, wrote in his first evaluation of summary writing that he could just learn from the lesson and did not need the writing. Piggy, a below basic Latino with high writing ability and some financial assistance, stated in his interview that he did not enjoy writing, though he did enjoy the social aspect of the writing process, including class discussions and discussions with his peers about the writing. He also recognized the benefits of writing in helping him remember the content and to visualize examples and their solution steps, techniques he felt he might use in the future. Student #3 said she would like to do problems instead, a sentiment echoed by student #81, who called herself Wendy for the interview process. Wendy, a below basic Latina with low writing ability who received free lunch, did not, however, repeat this opinion in her interview. She

did indicate at one point, though, that the examples were more important to her than the writing itself. She also that she did not like writing in math and that explaining in words was difficult even when she understood the mathematics, but indicated that the writing did help her and that she may continue to use the types of writing she learned in future math courses. The type of writing that seemed to be least helpful was self-monitoring and that the most helpful to her was procedural writing because she could identify steps she did not know and learn them at that time.

Other students also indicated that the writing in math was less helpful than working problems. Student #50 stated that words complicated things and that equations were better, while student #74 said all he needed was to be shown how. Student #75 also said that what helped him was to be working on what he needed to be working on, not writing it down. Student #47, mentioned above, perhaps stated the essence of this particular theme most accurately when he said, "All you're doing is writing; you're not working on it." His implication that writing is not really working on math is the fundamental misconception related to the theme of resistance to writing in mathematics.

Elaboration

A second theme that emerged from the data had to do with elaboration. At first, the teacher tried to encourage students to support all types of writing with either examples within the text, with reasons why a skill, step, or concept was important to remember, or to communicate their understanding or lack of understanding through examples. However, from the beginning of the study, many students resisted the use of any type of elaboration on their basic summary or procedural writing and their self-monitoring. As a result, the teacher decided to focus on only one type of elaboration, the use of examples embedded in text to focus students' attention and writing in one direction that could be encouraged with the whole group. Since the use of examples was a frequent topic in the classroom, a question about examples and their use in writing was included in many of the interviews. Even though participants continued to need much encouragement to use examples to illustrate their writing points, some students did find their use helpful. Piggy, a student mentioned above, often used and referred to examples in his writing and said in his interview that examples made it easier for him do similar problems later on. He only skipped the example when the work was easy

for him and he did not feel it would help. For Jeanette, a basic Latina with high writing ability and no financial assistance, using examples in her writing helped her remember and understand the content better because of the repetition they provided. Examples in the writing helped the math get “stuck in my head more.” Betty, a far below basic Latina with high writing ability and free lunch, said that including examples in her writing helped with what the writing means. Junior, a below basic Latino with high writing ability and reduced lunch, stated that the examples were necessary because they were what told him how to do the problems. For him, and a few others, the visual aspect of the example worked together with the written text. Jill, a basic white female with high writing ability and no financial assistance, echoed that opinion, stating that the examples helped her see what was actually going on and that it made her think, not just copy the teacher. Tamara, a below basic Latina with lowest writing ability and reduced lunch, also stated that examples helped her see how to do the problems. Cristiano, mentioned above, also felt that using examples in his text helped him figure out problems that gave him trouble. Another student, a far below basic Latino named Boise with low writing ability and free lunch, also felt that including examples in his writing made it more difficult because he had to think harder. Abigail, a below basic Latina with low writing ability and free lunch, said that the combination of examples and words were important to her and that she could see how the writing “fits with the work,” especially when she was writing about things she did not understand. Omar, a far below basic Latino with high writing ability and free lunch, is one of a few participants who did not see mathematics as just doing problems. He said that “...math is not just problems, it’s the writing and everything.” He saw the writing and examples as parts of a whole, and both parts were needed for complete understanding. Omar also had great insight into the use of examples in writing. Students were encouraged to make up their own examples in order to get them to think more deeply about the mathematics. Omar said: “...she told us, like we can make our own problems like. So try and make our own paragraphs with different numbers. So we could like understand it more, so like we won't have to be copying her or what she does, and it's better for us that we did it by ourselves because it helps us a lot. Because we, we understand how to do it so we can understand it better and like you get better on it every time.”

Though most students saw at least some benefit to using and referring to examples in their writing, there continued to be those who saw no benefit to elaboration of any kind. Miranda, mentioned above, said in her interview that using examples, especially going back to include examples in the test, was hard for her, and she did not like it. Students like Miranda continued throughout the study to make elaboration a challenge for the teachers in the classroom.

Results of Self-Monitoring on Curriculum

One of the most surprising themes that emerged had to do with changes teachers may have made in their teaching as result of identifying areas of need from student writing. It was not that this theme was not anticipated in the literature, but that it occurred even though the researcher tried to counter the effects on curriculum as a result of reading student work found in the literature. The discussion in several earlier studies with quantitative results that showed a significant positive relationship between writing and academic gain suggested that feedback from students' writing to the teacher may have had an effect on academic gain, making it difficult to tell whether it was the writing or the changes in curriculum for all students or individuals from the writing that accounted for the magnitude of the associations between writing and positive achievement gain. In this study, the researcher purposely tried not to allow the results of students' self-monitoring to influence the curriculum, choosing to read student blue book entries well after the content was covered in the class. Yet, several students mentioned the changes that may have occurred in instruction they received from the teacher reading their writing as a benefit of self-monitoring. Teacher observations indicate that instruction on the content was given to students during the writing process to meet their individual needs, as is ethically required, especially during summary and procedural writing. However, the feedback on the content that was given during the writing process would also have occurred if students were working on additional problems instead of writing. Since the purpose of self monitoring was for students to identify areas of strength and weakness for themselves, instruction was especially limited during self-monitoring, as teachers in the classroom encouraged students to write down the questions they had instead of having them answered on the spot.

Even with these measures taken to reduce the effect of feedback on achievement, a few students still identified feedback as a benefit of writing in math in their evaluations and in interviews. While the number of students who identified this benefit was small, the mention of it at all was surprising to the teacher. Some students assumed a clear cause and effect relationship between their self-monitoring and any subsequent help they got on the subject. For example, on the third evaluation, student #61 said that he preferred self-monitoring over the other types of writing because "...then the teacher knows what I need help with." Student #15, also in response to the question about whether self-monitoring had helped her learn the mathematics, said, "I think it [self-monitoring] does [help] because you(sic) helping the teacher at the same time so that they can learn and know(sic) what you understand and what you don't understand." Similarly, student #17 said self-monitoring gave the teacher a better understanding of what the class needed to work on, and Student #37 said that when she self-monitored and indicated a lack of understanding, the teachers then taught her more about that content. Tina, in her interview, said that, even though she had a difficult time writing down what she did not understand, "...when you tell the teacher that you don't understand it by writing this, she can actually help you before you take the test or do anything with it. So you can go back to it and understand it."

On the other hand, Student #33, Boise, recognized some kind of relationship between his self-monitoring and future help on content he did not understand, but drew no conclusions about cause and effect: Yes, [self-monitoring was helpful] because after sometimes I've put yes or no Ms. ____ explains it again or gives us a test review and it gets easier by the stuff we do." Student #31 was also somewhat vague about the relationship between her writing and the help she received later on: I think self-monitoring has helped me the most because u get to ask them what u don't understand and when they answer u you get it." It is not clear whether the help she got was at the time of the writing or in subsequent lessons.

Student #19 gave the clearest statement of what these students assumed to be true: "...if I don't understand it I write in my blue book and the teacher reads it and whatever I don't understand she helps me with it so yea it helps." However, the student also indicated that awareness of his own academic needs as a result of self-monitoring prompted him to seek help from the teacher: "Well self monitoring helps me a lot it makes me understand

what I'm doing if the problem is right or wrong if I don't get the problem I get help from the teacher and helps me understand the problem." This effect of self-monitoring on student awareness and subsequent search for help is a legitimate benefit of the writing, itself, not an unintended consequence. Student #28 also recognized this relationship between self-monitoring and learning: "Yes because it helps me to understand it better if I didn't understand the lesson it lets me write it out and review & ask questions about it."

While it was thought that all possible precautions were taken to limit the effects of students' self-identification of academic needs, it is possible that the teacher did pay attention to student concerns during the writing process and, either consciously or subconsciously adjust instruction to meet student needs. After all, that is what teachers are trained to do and, after 23 years of teaching, it is likely that this teacher may have done so automatically. However, it also likely she would have done the same if the activity had been something other than writing, as well. Finally, one of the interviewers mentioned to participants that self-monitoring might have provided the teacher with information about what students needed to work on. Those data were not included in these results.

Writing as a Reference

In her theory about writing, Emig (1977) stated that one of the unique qualities of writing is its concrete nature. Unlike speech, writing is immediately available for reflection and revision. In this study, a theme emerged as an unintended consequence of the physical nature of writing. From the earliest evaluations, students stated the use of their writing as a reference to help them remember the skills and concepts under study as a benefit, or potential benefit, of writing. As part of the classroom management system, the blue books in which the students wrote were kept in large interoffice envelopes that were usually available to students. In addition, students were encouraged to finish their writing at home if they needed more time.

Many students in the study referred to their writing as a reference, with some who said that if they forgot how to do a problem, they could look back to it in the blue book, and a few who said they could not or did not refer back to their writing. One student, (#6) said: "Yes it [summaries] has helped a lot because you can go back and see what you forgot so its a reference to help you remember." Some of the students, such as student #14 in one of her

evaluations, and Wendy in her interview, specifically mentioned the usefulness of the examples and notes they wrote in helping them remember or understand what they were learning later on in the class. When the interviewer asked Wendy if it was just the example she used to refresh her memory, Wendy affirmed that it was not just the examples that were of use, but also the writing itself. Tina said, "...when you write examples you can remember what you were doing before so you know what you're doing now, and you don't have to go looking for it in a book or something." Juke (#22) referred to his writing as a reference in both his evaluations and in his interview. In his third evaluation, Juke suggested open blue book tests, indicating the value he placed on his writing. In his interview, he stated that students could look back in their blue books at the examples to get a "good visual image" of the problems they had forgotten how to do and even wished the class was still writing so that he could use it as a reference when he encountered new problems that he couldn't figure out. A few students specifically mentioned using the blue books to study for a test. Cristiano mentioned this use of his writing in his interview and said he was using the procedural writing in science as notes. Omar (#20) mentioned the use of the blue books to study for tests in two of his evaluations, even suggesting to the teacher in one of them that students be allowed to study their blue books for five minutes before a test. Omar apparently did take advantage of his writing as a reference because in his interview he said, "Well, it, it helps you a lot because it like, it gives you step by step and then you could understand it more. Like, it's like if you're trying to do a speech you have to memorize it and you read it all over and over again. And then, like, when we did the test we, we had some time, we had time to look under back to our blue books and then see what, what we learned and like how to do it. And then like you get the, you get everything." Another (#8) referred to the blue book as a guide to how to do the problems and said that he could use his blue book to look back and see what he had learned. Taylor, in his interview, affirmed that his blue book was a good source of reference for him, as well, especially since, as the interviewer pointed out, it was written in his own words.

What was especially interesting about this theme was that a few students assumed the blue books were not available for their use once outside the writing time in class. Some students may have taken responsibility on themselves for not referring to their writing. Student #31 said she never had time to look over the blue book and another student, #27,

stated, “No, because I never go back to it and also I forget sometimes why I wrote that.”

Student #9 also said on his second evaluation that summary writing was not helpful because he never looked at it. Of course, these students may also have been implying that they were not given the time to refer back to their work during class. Student #39 did not think he got them back to look at it after the writing was done and student #75, Gunther, said he did not look it over after writing it and said in his interview that he did not get it back after writing in it. He suggested to the teacher that students should look at their blue books more.

Student #69 also clearly felt the books were not available for students after writing: “...we cannot keep the book or study from them.” In this student’s opinion at the beginning of the study, if the writing was not to be used as a reference, the writing was not useful or helpful, though his opinion changed in subsequent evaluations. Still another student (#64), Tamara, said in her first evaluation that she could look in her blue book, then said in her second evaluation that she did not like self-monitoring because it was “...still hard to remember stuff when we don’t even look at it,” then said that procedural writing did help her because when she forgot something she could just look at her blue book. This student’s confusion, and the reason for the conflicting data surrounding this theme, may have been the result of the unintentional lack of direction and time given for specifically using the blue books as a reference. It seems clear that many students assumed writing would be used as a reference and expected their writing to play that role in mathematics.

Grades

Like resistance and elaboration, this theme was also anticipated in the literature. Many researchers made the decision not to grade students’ writing in order to reduce the stress of writing and create a less restrictive writing environment. This researcher, however, having worked with struggling mathematics students for many years, recognized the need for some sort of reward/consequence system for her students. She believed that, while most students would write, they may not put as much effort into the writing if they were not to receive points toward their grade in return. In addition, there are a few students who would have refused to write if their writing had not been a part of their grade for the class. As a result, points were awarded for completeness only, not for correctness of content. The writing grade comprised approximately 5% of their entire grade. During the writing process

throughout the study, several students had to be reminded that to receive full credit, they needed to include elaboration with all types of writing or write about what they did not understand as well as what they understood during self-monitoring. In his third evaluation, Juke mentioned that he felt that self-monitoring only helped the students' grades, and student #78 said on his first evaluation that summary writing was not helping him because he was just writing what he was supposed to write. Interestingly, the same student said on his second evaluation that summary writing had helped him to remember what he summarized, then said on his third evaluation that no type of writing helped him. In a group interview, a few students from Group 1 commented that they thought the writing process would have been better if there had not been a grade attached to the writing. When asked how the teacher could improve on the writing process the next year, another participant said that writing should not count towards students' grades because it made students worry about having the "exact information." Duke also mentioned the "stress issue" and how writing made him worry about his grade in the class, and Betty agreed that the grade causes students to worry more about their grade than about their writing, a very insightful comment. Most Wanted said that there was no best part about writing because it affected his grade. The interviewer then asked whether the students would have done a good job on their writing if it did not count toward their grade. Though several said they would have, Mike said he probably would not have done a good job because he did not like writing. While not grading students' writing in mathematics may be an ideal for which to strive, Mike's honest confession supported the teacher/researcher's decision to assign a minimal grade in order to encourage students who do not like to write to participate in an activity believed by many to support the learning of mathematics.

Writing Without Understanding

One of the themes that seemed to emerge slowly throughout the study was the students' inability to write when they did not clearly understand the mathematical concepts under discussion. Hints of this theme began to emerge in the first evaluations. While some students participated in the writing without encouragement, and others resisted writing because they just did not like to write or felt it was a waste of time because they already knew the mathematics, or were resistant to schooling in general, a few students who tried to

write just copied what was on the white board, such as fragments of sentences to remind students of the important points in a lesson. In the first evaluation, two students wrote that understanding the mathematics was a prerequisite to being able to write about the lesson. Miranda said that self-monitoring did not help if she needed help with the problem itself. In all three of her evaluations, student #57 was clearly frustrated with the process. In her first evaluation she said that writing in the blue book did not help her learn because "...I really didn't understand it but the teacher asked the class if they really understand it and everyone did but I didn't say nothing because I somehow just think I cant do it, I just say I did understand it and I just go to my notes and copy them into my blue book." In her second evaluation, she said that self-monitoring did not help because she said what she could "...without even meaning it!" She went on to say that she did not know what to write because she did not understand the lesson of the day. This comment came from a Latina who was far below basic in mathematics but with relatively high writing ability. She struggled with the mathematics on a daily basis. In her third evaluation of writing in the class, she again said that writing did not help her because, "...most of the time I don't get the 'lesson of the day' and I have to write about what I learned and I didn't learn." For this student, writing must have been a very frustrating experience. By the third evaluation, a few more students were coming to the same conclusion. Student #3 said that writing sometimes helped *unless she did not understand it*, and Jill said that summary writing had not helped because, "...we had no idea what we were writing about." In the Group 3 interview in which Jill was a participant, she said that the worst part of writing was that some people did not understand that they were writing. When asked by the interviewer what would happen then, Jill said that there would be no point to the writing and that the writing would not be useful. The group interviewer had already told the teacher after the Group 1 interview that one thing that was clear to her was that students felt they could not write if they did not understand the mathematics involved. During the Group 1 interview, Duke, a white male with low writing ability but basic prior knowledge, expressed his concern over getting a lower grade when he did not understand "the basic procedures" or basic lesson of the day. Another student countered that that was what the writing was for, to tell the teacher what he did not understand. Duke responded by saying that if you do not understand, you do not know what to write. The interviewer then rephrased Duke's concern: "I'm going to get a grade for this so

I want to write it the way it's supposed to be." And you're [Duke] saying, "If I don't understand it, how can I write it the way it is supposed to be?" Duke agreed that she had stated his case accurately. The student arguing with Duke was clearly referring to self-monitoring, while Duke may have been referring to either procedural or summary writing, or both. It seems that some students, those concerned about their grades and those who just wanted to participate in the writing process, were frustrated with having to write when they did not understand the mathematics. While students can probably write about a story they have read, even though they did not completely understand it, writing in mathematics, especially informational writing like procedures and summaries, may be different in that even students with relatively high writing ability or mathematical proficiency may experience frustration when trying to write about mathematical concepts they do not fully understand. Self-monitoring was, for some, the answer to the problem of not being able to write about what you do not understand. In her interview, Abigail, a Latina with low writing ability and below basic prior knowledge, agreed with the student arguing with Boise above and wrote that self-monitoring did help her understand better because if she did not understand a lesson, it let her write it out, review, and ask questions. While the student in the group interview with Boise assumed the teacher would read his or her questions and answer them later in the class, Abigail gave a more accurate and concise statement of the function of self-monitoring, to lead students to know what they needed to work on and get outside help for themselves if needed.

Inability to Express Thoughts

One of the most persistent themes in the study had to do with students' inability to identify and express how the writing had or had not helped them. During the data collection phase, when students were writing their evaluations of the types of writing they had done, the teachers in the class were constantly encouraging students to try to explain what they meant by their "Yes, it helped," or "No, it did not help" answers. Some guidance, perhaps leading, was given in the evaluation prompts, which each asked if the type of writing they had done had helped them "think about, learn, or remember the mathematics." Many students found it difficult to say how the writing had helped them or why the writing had not helped them, which made it difficult to quantify the benefits of writing as perceived by the students. Some

were very clear, including examples of how the writing had helped. For instance, student #58 wrote that summary writing had helped them remember why it is important to have the same denominator and reduce the answer if needed when adding and subtracting fractions, while Most Wanted said summaries helped him remember proportions and equations, and student #51 wrote that self-monitoring helped her understand how to change fractions to percents. Many students simply wrote that the writing helped them remember, understand, or learn the math, but they could not seem to say how the writing helped them, either orally, as they were writing their evaluations, or in writing. Wendy stated in her interview that the hardest part of writing in mathematics was that "...you might know how to do it, but you don't know how to explain it in words."

Several other students also said writing helped them with the meaning of the content, to understand, make sense of, or learn the mathematics. These answers are difficult to categorize and quantify, and yet they shed light on the perceived benefits of writing in mathematics. For instance, student #16 said "...I get things better" than when someone tried to tell him how to do the math, and student #18, who was often absent, said that procedural writing helped him understand the problems, and it helped him learn about it. Student #13 also said that summary writing helped her "get more things" that she did not "get" before. Student #1 wrote in her second evaluation that when she started writing summaries, she started to "learn more about the mathematics." Student #63 said that summary writing helped because it sometimes made more sense when she wrote it down, while student #3 said that when she wrote it down in the blue book, she could actually see how to do a problem, even when she did not understand the directions from the lesson. Student #7 wrote that procedural writing had helped a little because she got to "learn how to do it," and that she preferred procedural writing over summary writing because it made her think and go through all the steps to get the answer." Student #56 wrote that writing the steps(procedure) made it easier for him to figure out the problems, while student #52 simply said on his last two evaluations that step by step helped him understand and remember the content. Student #29 said that summary writing helped her because she could understand what she was doing and learning, while student #36 said that summary writing was helpful because it explained things the class was doing. Student #52 said that procedural writing reminded her about more things and that she was "kind of understanding" as a result. Several students, such as Jill, also recognized the

use of their own words as an aid to their understanding, though they did not, or could not, say how. Student #39 wrote about self-monitoring that writing made him think better and harder, and student #50 wrote that self-monitoring reminded her about what she had been learning, possibly referring to the property of writing as an object on which to reflect, or a process that leads to reflection. In his second evaluation of procedural writing, student #28 said that he, too, reflected on his work as part of the process prior to writing and that his writing helped him understand how to work on new problems. In the same class and on the same evaluation, Jill stated that writing made her think about the problems and understand the steps, again indicating that reflection was part of the writing process. Furthermore, in her interview, Jill said that by writing it down, she had to think for herself, not just copy what the teacher wrote down. Student #6 wrote that when she was writing procedures, she was thinking how to solve those types of problems.

Writing and Remembering

Writing as an aid to memory was also a prominent theme. Many students wrote that the writing they did, especially summary and procedural writing, helped them remember the mathematics. A few students supported their affirmation of writing with specific examples of what they remembered. One of these students was student #13, an African American female who was far below basic in mathematics and had low writing ability. This student was often absent and failed the class both semesters. When asked whether procedural writing helped her, though, she responded with an unequivocal “Yes” and gave examples of how the writing had helped her remember fraction, decimal, and percent equivalents. Student #8 said that procedural writing helped him because he could not forget about the content we had covered in class, and student #21 wrote that writing procedures had helped him a little because before he started writing procedures he used to forget the mathematics. Student #6 said that self-monitoring really helped her remember everything that she wrote and that by writing, she did not think she would forget the material. In her second evaluation, student #27 said that writing only sometimes helped because some “...don’t get in my head.” In her final evaluation, Wendy chose procedural writing with elaboration (reasons the steps are important) as helpful to her because “...writing down a mathematical problem or idea is very good because it helps you remember the problem better.” She also recognized that reading,

writing, and mathematics are all connected, a profound insight for one whose peers think writing is only for English class.

Two ways writing helped students remember the content were visualization and repetition. Student #38 said that procedural writing helped her remember the math because she repeated it over and sometimes it “stayed in her mind.” Student #39 simply said that if he wrote down the steps in a problem he would remember how to do the problem. Jeanette said in her second and third evaluations that she knew it better the more times she wrote it down. When asked in her interview if the writing helped her visualize the mathematics, she said that she saw the numbers in her head, “Because you write it down and you know like, I write down something, explain it, and I have it in my head and like doing it in my head.” She affirmed that it was through the writing that she could picture how to do problems in her head because she was writing it down. In the same interview, Jeanette had a difficult time explaining that the reason the examples helped her was that she repeated the work more times and the math got “stuck in her head more.” At least two participants in the Group 1 interview said that writing helped them remember the content because they repeated it, writing it down after learning it in the lesson. Student #19 said something very similar on his first evaluation of procedural writing when he wrote: “Yes [procedural writing helped me] because it actually gets stuck in my head...” Student #72 said that procedural writing, explaining how to do problem, helped her memorize how to do the problem, while student #12 wrote in his first two evaluations that procedural and summary writing helped him remember the mathematics and painted a clearer picture in his mind, again referring to visualization as a beneficial outcome of writing for some students. Student #37 said that procedural writing helped her remember the steps of other problems, indicating transfer of skills from one situation to another.

Visualization also helped at least one student understand the mathematics better. In the Group 3 interview, he said that the best part of writing for him was that, “...you could see the see the math problem and you could write about it, ...like you visualize it and you write it down, you understand it better.” In the Group 1 interview, one of the participants said that incorporating examples in their writing helped him visualize how to do the problem in different situations, again implying the transfer of skills form one situation to another.

In summary, several themes related to the types of prompts used in the study and their effect on student learning according to the students themselves were revealed through qualitative analysis of student evaluations of the types of writing in which they engaged throughout the study and student interviews after the experimental phase was complete. In addition, all three types of writing were considered effective by a variety of students at varying times for various reasons. The next section of this paper will present the researcher's summary of both quantitative and qualitative data, as well as her conclusions drawn from the data, as well as suggestions for further research in writing in mathematics.

CHAPTER 5

CONCLUSION

The purpose of this study was to determine the effects of different kinds of writing on the achievement and attitude of eighth grade students who struggle in mathematics. This study explored three research questions:

- Does the type of writing prompt affect student achievement in mathematics?
- Does the type of writing prompt affect students' self-concept of ability in mathematics?
- Do students' self-concepts of ability in mathematics affect achievement in mathematics?

Students in each of the teacher-researcher's three Middle School Algebra classes (a course created to prepare eighth grade students for Algebra 1-2 in ninth grade) comprised a treatment group and were asked to respond to three different types of writing prompts: summary, procedural, and metacognitive (self-monitoring) over the course of three 5-week units of study. For instance, Group 1 responded to summary prompts during the first unit of study, Group 2 to procedural prompts, and Group 3 to self-monitoring prompts. During the second unit of study, Group 1 responded to self-monitoring prompts, Group 2 to summary prompts, and Group 3 to procedural prompts, and so on. Students were taught how to respond to each type of prompt through direct teaching and modeling on the first two journal, or blue book, responses, then were asked to write approximately twice a week for a total of ten responses for each unit of study. The first unit of study included skills and concepts related to rational numbers, including probability, percent, fractions, and decimals; the second unit of study was about graphing, tables and equations; the third unit was about proportions. Achievement was measured by teacher-created pre-and post-tests examined by a professor of mathematics teacher education, who gave input and found the tests valid for content.

In order to measure students' self-concept of ability in mathematics, students were asked to fill out the entire Minnesota Mathematics Attitude Inventory (MAI) at the beginning and end of the study, and to respond to the 8-item subscale of the MAI for self-concept of ability in mathematics at the end of each unit of study. In addition, students were asked to

evaluate and compare their experiences with the different kinds of prompts at the end of each unit of study. After the treatment period, 21 students volunteered to be interviewed individually about their writing experiences in mathematics during the study, and all but one of those students were available to participate in group interviews, one for each treatment group. The majority of the individual interviews were conducted by two former mathematics teachers, while one individual interview was conducted by a mathematics teacher from the same site as the researcher, and the three group interviews were conducted by one of the former teachers who also interviewed students individually. The interviews themselves were semi-structured, giving the interviewer the freedom to follow students' responses and interviewers' curiosity wherever they led.

SUMMARY OF FINDINGS

Quantitative analysis included descriptive analyses and comparisons of means using ANOVAs and corresponding non-parametric tests of the data related to the units of study and groups, as well as data related to each research question, and linear regressions related to each research question. Analyses revealed statistically significant differences between the rational number unit and the other two units. Specifically, students had more prior knowledge of the content for the unit about graphing, tables, and equations, according to pre-test scores than about rational numbers, yet statistically significant lower achievement scores on the post-test. Analyses of the data by group indicated that treatment Group 3 had proportionally more females and Latinos than the other two treatment groups, was academically less prepared than Group 1 with regard to prior knowledge, and less academically prepared than either of the other two groups with regard to writing ability or pre-test scores. Group 3, however, did make significantly greater achievement gains relative to their pre-test scores than the other two groups.

With regard to the first research question, descriptive and preliminary inferential statistics, such as ANOVAs and corresponding non-parametric tests, indicated no statistically significant differences in achievement between the treatment groups. In other words, no specific type of prompt was associated with a statistically significant achievement change when compared to the other prompts. However, multiple regression analysis using pre-test scores, self-concept of ability pre-test scores, change in self-concept of ability, prior

knowledge, writing ability, and the demographic variables of ethnicity, gender socioeconomic status, type of prompt, type of unit, and group as independent variables, and achievement change measures (see Chapter 4) as the dependent variable, revealed that, while controlling for all other variables noted, the use of summary prompts was positively associated with achievement gain at the .05 significance level in a model that explained approximately 40% of the variance in students' achievement scores. However, the association was small ($\beta = .134$) and the effect was found for only one of the four measures of achievement.

Descriptive and preliminary inferential analysis of the data related to the second research question (Does the type of prompt have an effect on self-concept of ability in mathematics?) by unit indicated that there was an upward trend in self-concept of ability prior to each unit of study, and that there were significant differences in self-concept of ability pre-scores between Unit 1 and the other two units of study. While the pre-scores were rising, there was a downward trend in the change of self-concept of ability from unit to unit. Analysis by group indicated that males had a significantly higher self-concept of ability than females prior to treatment, White students had a significantly higher self-concept of ability than Latino students, students with the highest socioeconomic status had a significantly higher self-concept of ability than those with the lowest self-concept of ability, and those with the highest level of prior knowledge had a significantly higher level of self-concept of ability than those with the lowest level of self-concept of ability. There were no significant differences in changes of self-concept of ability by group as a result of treatment. In addition, multiple regression analysis, with change of self-concept of ability as the dependent variable and achievement change, achievement pre-test scores, self-concept of ability pre-test scores, prior knowledge, writing ability, ethnicity, gender, socioeconomic status, types of prompt, type of unit, and group as independent variables, indicated no significant association between the type of prompt and change in self-concept of ability. However, self-concept of ability pre-test scores, achievement pre-test scores, and achievement change measurements were found to have small to moderate positive associations with changes in self-concept of ability, while gender ("maleness") had a small negative association with changes in self-concept of ability.

Results of multiple regression analyses related to self-concept of ability suggested that both self-concept of ability pre-test scores and changes in self-concept of ability had small positive associations with achievement on two different measure of achievement in this study.

SUMMARY OF EVALUATION PROMPT RESPONSES

Following the post-test for each unit of study post-test, students were asked to evaluate the effectiveness of the type of writing they had been using, and, after the second unit, to compare the types of writing they had done so far. In general, a majority of the participants in the study (over 50% in each case) gave unqualified positive responses to their writing experiences with all three types of prompts. Students wrote that the prompts helped them remember, learn, and understand the mathematics, and that their writing served as notes to help them study. Those who did not find the types of writing helpful generally felt that either they did not need the writing because they understood the mathematics well enough, or that they did not understand the mathematics well enough to write about it.

The most popular type of writing was procedural. Students who chose this type of writing felt that the step-by-step nature of procedural writing and breaking down the problem was the best way for them to learn from their writing. Metacognitive writing was the second most popular type of writing, though summary writing was more popular than self-monitoring in Group 2. Students who selected metacognitive writing as their first or second choice wrote that self-monitoring helped them focus on what they did not understand so that they could either study or get specific help from the teacher. Some students thought their writing would provide feedback to the teacher that would result in focused instruction on what they needed most. Summary writing was the least popular type of writing for two out of the three treatment groups. Students who preferred summary writing said that summaries helped them focus on and remember important information.

SUMMARY OF QUALITATIVE RESULTS

Qualitative results in this study came from teacher observations, student responses to evaluation prompts, and individual and group interviews. Results were organized by type of prompt (procedural, summary, and metacognitive, or self-monitoring) and by the themes that emerged through the constant comparison method of analysis. Themes included:

- Resistance
- Elaboration
- Effects of self-monitoring on curriculum
- Writing as a reference
- Grades
- Writing without understanding
- Inability to express thoughts
- Writing and remembering

Students resisted writing in a variety of ways. Some students just did not like to write, did not like mathematics, or did not like school in general.

DISCUSSION

In their article describing a framework of predictors of academic achievement in math and science, Byrnes and Miller (2007) propose three categories of predictors of high achievement: opportunity, propensity, and distal. Opportunity factors can be provided in school or outside of school and include any content to which the learner is exposed. In this study, the writing activities that comprise the treatment are the opportunity factors that are being examined and tested. Propensity factors, on the other hand, are "...factors that relate to the ability or the willingness to learn content once it has been exposed or presented in a particular context" (p. 601). Propensity factors in this study, then, include cognitive factors, such as students' writing ability scores and pre-test scores, and motivational factors, such as measures of self-concept of ability in mathematics prior to treatments. These two types of predictors, opportunity and propensity, are considered proximal factors in achievement. The third type of predictors identified by Byrnes and Miller are the distal factors, which include socioeconomic status, student and parent expectations, and prior educational experiences. The distal factors in this study include socioeconomic status and prior knowledge. In this study, distal and propensity factors had the most effect on both achievement and attitude, while the opportunity factors involved seemed to have had little or no association with achievement or attitude.

While Byrnes and Miller (2007) point out the complexity of predicting achievement in mathematics and science, this study points out the complex relationships between curriculum, instruction, and student expectations and attitudes in writing to learn in

mathematics. The discussion that follows describes the teacher/researcher's experiences in implementing writing to learn in mathematics while analyzing its effects on achievement and attitude, especially as those experiences relate to the intended curriculum and planned instruction, and how they relate to student expectations and attitudes.

One of the complexities that arose was the relationship between the planned curriculum and instruction as defined by the methodology of the study, the writing prompts, student needs and expectations, and the classroom environment. While the study called for the use of one prompt for a given period of time, it became clear from the beginning of the study that the type of prompt should be determined by the curriculum and, after becoming familiar with the different types of writing, by the student. Summary prompts were especially effective and accepted by students, according to teacher observations, when the curriculum addressed relatively broad concepts, such as graphing, time and distance graphs, probability, or solving one-step equations. Procedural writing was more effective when specific procedures, or skills, were emphasized, such as solving a specific type of equations, adding or subtracting fractions, finding percents, discounts, or taxes, and other algorithms. Summary and procedural prompts, then, should be determined by the curriculum. Metacognitive prompts were more useful, when students still had some questions about the content, when the curriculum was not familiar, new, or extended beyond their current understanding. Several students who said that self-monitoring was not helpful and resisted writing stated that they already knew the content and did not need to write down any questions. While some of those who resisted would only ask oral questions, expecting immediate feedback so that they did not have to reveal that they did not know something in writing (an unsubstantiated speculation), others may really have understood all that they were asked to know. They could have gone beyond the question, but that is not characteristic of the population under study. A different type of prompt may have been more beneficial, such as a summary or procedural prompt with elaboration.

Each of the types of prompts used in the study was helpful under the right circumstances. Summary and procedural prompts were most beneficial and fit naturally with the curriculum when students were at least fairly familiar and comfortable with the concepts and skills they were studying. Metacognitive prompts were most beneficial when students were still struggling somewhat with the content and could work out their questions on their

own. The use of one type of prompt, regardless of content, created a tension in the writing process, as well as the instructional process, as students tried to write in a form that was not a natural fit to the curriculum or to their level of understanding. One conclusion, then, that can be drawn from this study is that the type of writing in mathematics should be determined by the curriculum and the needs of the students.

Writing instruction was also complex and related to not only the curriculum, but to the academic and affective needs of the students. Each prompt was introduced to students first through direct instruction, then as a group, then on their own, though students could work with others after a few minutes of individual reflection. Even in direct instruction, it was difficult to keep the writing simple, yet address the most critical connections to related skills and concepts. The artificial use of an ill-fitting prompt was especially difficult. For instance, when trying to introduce procedural writing, the teacher found it difficult at times to include all the various relationships between the variables involved in the concept and the actual algorithm. When introducing procedural writing, for example, the first prompt asked students to describe how to make a coordinate graph, or scatterplot, from a table.

Procedurally, students were asked to copy the table, draw a coordinate plane, then graph each point. However, related skills and concepts included determining and numbering each axis with appropriate scales, knowing which axis is x and which is y , and knowing what a coordinate pair is and how to plot it on the plane, concepts and skills not familiar to several students in this population despite several years of exposure to the content.

While procedural writing may have been the easiest for many students, simply describing what they were doing, on-the-spot formative assessment would have indicated that summary writing may have been more beneficial for many of those who still struggled with the concepts involved, and self-monitoring may have alleviated some of the students' frustrations with a curriculum with which they were familiar but not yet comfortable even after several years by helping them sort out the sources of their confusion. While the researcher had to continue with the prescribed methodology, it became apparent that, ideally, students needed training in the use of multiple types of writing and to have the experience necessary to be able to choose the appropriate type of prompt that would help them the most in any given situation.

One of the somewhat surprising phenomena that surfaced during the study was the change of mind, of attitude, many students had towards writing in mathematics. Many students stated in their evaluations and interviews that, even though they disliked writing in general, or writing in mathematics specifically, they came to see that writing had helped them in some way. Even though quantitative results indicated that no particular prompt had a greater effect than the others in the study, student self-reports indicated that writing did help them learn, remember, and understand the mathematics they were studying. For some students, procedural writing helped them sift through the steps of a problem, whether a simple algorithm with few steps, or a more complex, multi-step procedure. Adding a layer of elaboration seemed to keep both the mathematical and the writing task from becoming trivial. For others willing to risk being wrong, thinking about their thinking, self-monitoring, was most useful, helping them find their areas of strength and weakness. Summaries helped some students get the big ideas and to see how ideas were applied to specific problems and could be applied in different settings. While they did not particularly enjoy the writing, many students found more than one type of writing useful in the learning of mathematics. Even the evaluation prompts, considered simple data-gathering tools for research purposes only, provided some students with a much-needed outlet for expressing their feelings about their experiences in mathematics. The insight of these students, who recognized the usefulness of a tool such as writing, even though not associated with pleasant memories or fun times, revealed a maturity as learners that the teacher/researcher would not have known existed. These students showed that writing in mathematics can be useful to both the teacher and the student when proper instruction and encouragement is given. Furthermore, it became apparent that no one type of writing is sufficient. A diverse population of students calls for the need for diverse experiences, both mathematically and in writing.

In order for students to be able to choose the type of writing that will help them the most with the content they are experiencing, teachers need to be familiar with each type of prompt under various circumstances. While the researcher in this study attempted to think through each prompt and the curriculum to which it was attached, much was learned by using the various types of prompts with different content. Care must be taken to choose the best kind of prompt, or prompts, with the curriculum, especially when introducing the different types of writing. Much insight can be gained through evaluation prompts, as well. However,

just planning and thinking through the curriculum is not enough. Just as students must be willing to take risks in order to learn, teachers must also be willing to take risks and weather the complaints they get when introducing writing to learn activities in the classroom. This study provides evidence that writing can be an effective learning tool, not just another type of formative assessment or outlet for student feelings. The students themselves provided the evidence that writing helped them learn and, perhaps, even have a better attitude toward mathematics.

Another of the many complicating factors noted in the qualitative results in this study was the inability of students to write when they did not understand the content. While providing a variety of prompts, especially metacognitive prompts, from which to choose might alleviate some of this problem in day-to-day instruction, it seems that this problem may be especially relevant to mathematics, where the content, and understanding, builds from concept to concept and skill to skill. In other disciplines, such as English or history, students may have more of a network of connections, or schema, from their everyday lives upon which to build. If they read a story in English, or a passage about an historical event or person, students have more experience with stories, places, and people than they do with the more abstract properties of number, algebraic concepts, and algorithms associated with 8th grade mathematics. Most of the time, students have connections between people, places, actions, or even emotions to which they can, at some level, relate. For some, though, what happens in mathematics is like magic. Things change and happen, especially in algebra, for no apparent reason, and seemed not to have any connection to the next situation, the next problem. Unfortunately, previous mathematical experiences, whether at home or at school, have not been sufficient to build the connections they need to accommodate new material or to even recognize that there are logical, mathematical relationships and principles at work, so that extensive scaffolding is needed to create these connections and networks.

While the remedy for the situation is as complex as the condition, writing can make a unique contribution to that scaffolding, especially when elaboration in many forms is required, asking students to build connections to at least one example, or to relate their new learning to previous examples. Writing can also function as a unique form of feedback to give teachers the information they need in order to diagnose such a problem. Students who cannot write about the mathematics may have no knowledge network, no schema, into which

they can integrate new knowledge. As a result, the new knowledge is useless and simply frustrates both the learner and the teacher. Writing gives the teacher specific information about the kind of scaffolding and mathematical experiences these students need to build the necessary knowledge frameworks. These experiences may be as simple as sitting down to clear up confusions and helps students make connections for themselves, or as complex as providing students with fundamental experiences, perhaps with appropriate manipulatives, that support the learning that has already taken place for other students. At the same time, teachers can gain insight into the kinds of knowledge networks students have built from their writing. When students can make up their own examples, relate the content to their examples, and even extend or speculate about future applications of the skills and concepts they have learned, teachers may conclude that these students have robust schema and are ready for new material. A few students in this study exhibited this kind of knowledge network and enjoyed making up silly, but appropriate, examples.

As mentioned above, one of the most interesting phenomena that came to light during the study was the resistance of some students to write questions they had during self-monitoring. Most students were willing to write what they knew, though connecting their knowledge to a specific example was a challenge for them. However, when students were asked to write about what they did and did not know after a lesson, a few students could not seem to write the very questions they asked the teacher orally. Several of these students were asked to participate in the interviews following the experimental portion of the study, but all of them chose not to participate. As a result, only speculations are raised as to why these students found this aspect of self-monitoring so difficult for them, with no quantitative or qualitative evidence to support those speculations. Students who could express their questions in writing tended to ask oral questions and expect an immediate answer from the teacher. Many students in the American education system, from Far Below Basic to Advanced, expect the teacher to give them answers to their questions, not encourage or guide them to answers on their own. When students in the study were told to write their questions, they seemed stunned that the teacher would not simply answer their question. The teacher's response that they write their question and think about it, perhaps review the lesson notes and examples, was often met by a stare and silence. Some students still did not write their questions, though a few students did respond positively, wrote their questions, and even

answered them for themselves. At times, though, students became frustrated and implied that the teacher was not doing her job, indicating an expectation that the teacher was the giver of knowledge and they mere recipients, not active, responsible learners. While one of the purposes of self-monitoring is clearly to help students identify and clarify areas of need, it is also a way to get students to think more for themselves, to become more engaged in their own learning, and to reduce the role of the teacher so that students become more independent learners.

While expectations of the role of teachers and students in the learning process may have been one reason students would not put their questions in writing, it is also possible that some students did not want to risk showing ignorance. Many students are reluctant to show any weakness in their knowledge or understanding, especially in mathematics. When asked to present solutions and thinking processes to the class, students at all levels of ability tend to want to check to be sure their answer is correct before sharing their ideas with their peers. In second language learning, there is some evidence that people who are willing to take risks are more apt to learn a language than those who are not (Bang, 1999). The same is true for mathematics, especially since mathematics, for some, *is* like a second language. Recent pedagogical movements encourage making mathematics classroom environments a safer place for students to become risk-takers, to make their thoughts public in order to promote the social construction of knowledge. For many students, this is still new territory, and they need to be encouraged to be willing to be wrong, and to learn from their own misunderstandings. Writing, especially self-monitoring, where the learner is asked to identify his own weaknesses in understanding, can be a first step to becoming an academic risk-taker by providing first a safe place to ask questions, then to be encouraged to ask questions and share uncertainties in understanding more openly later on.

It was ironic that summary writing, the least favorite type of writing in this study, may also had the only statistically significant effect on achievement. The fact that several students mentioned that summary writing was difficult sheds some light on the problem with summaries. Students struggled with summary writing from the beginning, failing to recognize the concepts, the big ideas of the lesson, and then to put them into words. Far more scaffolding was needed for this type of writing than for the other two types of writing. For all types of writing, some scaffolding was given for the first two or three prompts—ideas and

sequences of steps on the board generated by the students themselves. After only the first two or, perhaps, three prompts, students were able to engage in both procedural writing and self-monitoring on their own, with some exceptions noted above. However, scaffolding for summaries extended well into the unit of study, as students, when asked to write about the big ideas of a lesson, gave indications of frustration, and little writing was produced. Even when major concepts were written on the board, students were hard-pressed to relate them to a given problem, let alone make up a problem of their own to illustrate their understanding.

While summary writing may be the most difficult for students to learn, its benefits may outweigh the time it takes to teach well. Many students preferred procedural writing for the simple reason that procedures “break down” a problem. These students are especially able to work with smaller steps, with concrete problems and materials. They are very comfortable working with particular types of problems, such as one- or homogeneous two-step equations. However, they often lack the ability to generalize, a key component of algebraic understanding, and have trouble applying general properties of number and equality, even when their experiences have included the generation of those properties. Educators have been struggling for years with bridging the gap between concrete reasoning and abstraction and have not yet come up with an effective tool that can be utilized with a majority of students. Algebra students continue to fail the course in great numbers at all levels. Writing summaries may not be the complete answer to this problem, but it is another tool mathematics educators can use to give students more experience with generalization and looking beyond the specific to the general.

For the most part, the study was conducted as planned in the methodology. However, the use of elaboration was a constant source of frustration for both students and teacher in this study. While the teacher recognized the need for elaboration in order to provide more depth to both the mathematics and the writing, what was first envisioned was not carried out. Elaboration can take many forms, including references to examples, mathematical justification of steps taken, and extension of the problem to new and different situations. However, just as the metacognitive prompt gave way specifically to self-monitoring, elaboration gave way to the use of examples to illustrate important ideas, steps in an algorithm, or mathematical understanding and non-understanding. At the beginning of the study, a choice was given to students that they could include an example within the text of

their writing, or they could give a mathematical reason for each of the steps they took in a procedure, or for the ideas they chose as most important in their summaries. It soon became clear that students were somewhat overwhelmed by just the task of writing and that choices had to be limited in order to model complete responses and keep the amount of writing time reasonable.

The results associated with self-concept of ability were also interesting and again emphasized the complexity of learning and the many variables, distal, proximal, and opportunity, that affect students' achievement and changes in self-concept of ability in mathematics. It is not surprising that distal factors related to prior knowledge were positively associated with changes in self-concept of ability, not that students with low pre-test scores may have had large gains in self-concept of ability while those with high pre-test scores on a given unit may have had much smaller gains, or even losses, in self-concept of ability for that unit. What is of interest is that, even though none of the prompts was associated quantitatively with self-concept of ability, either positively or negatively, student self-reports indicated that students did feel more able to do the mathematics than in previous years. While this rise in self-concept of ability might have come from factors other than writing: it is significant that the students themselves attributed their changes in attitude to the writing, itself.

What was somewhat surprising is that boys' self-concept of ability scores were moderately positively related to changes in self-concept of ability. Though boys are associated more with a preference for math and science and girls with a propensity for verbal expression, the results of this study show that, especially for boys, writing in mathematics may help increase their self-concept of ability in mathematics. It is interesting to note that more boys than girls self-reported confidence in mathematics, so much so that they did not think writing was any help at all because the math was so easy. Furthermore, in the interviews, it was boys who said that being able to express how they felt about math and writing was important to them. Writing may give males an outlet for expression, both informational and affective, that they do not feel they have otherwise in the mathematics class environment. Perhaps they feel as though they are supposed to be better at math than girls and do not feel free to express these thoughts in front of girls.

CONCLUSIONS

Several conclusions can be drawn from the data gathered and analyzed in this study. First, distal factors and proximal factors, such as prior knowledge and socioeconomic status, affected achievement and attitude more than opportunity factors, such as the type of prompt. Second, student self-reports indicate that writing in the way described in this study did help student learn mathematics and change their attitudes toward writing in math, and, if feeling they could remember or understand the mathematics is any indication, increased their self-concept of ability in mathematics. Third, the relationships between writing, instruction, achievement, and attitude are interrelated in complex ways that vary from class to class and student to student. Writing in mathematics must reflect the diversity of the needs of students and the nature of the curriculum, which includes a variety of writing prompts.

IMPLICATIONS FOR INSTRUCTION

While mathematics teachers often regard writing as a separate discipline, this study shows that writing can be a useful tool for students to remember and understand mathematics. Instruction in the use of the various prompts took some instructional time, but the time was well spent if students gained insight into their own learning through self-monitoring, were able to make connections between procedural steps and mathematical properties, or were able to build more robust and longer-lasting knowledge networks. Furthermore, students not only gained academically, but were able to identify the personal benefits of writing activities for themselves. Once students are trained in different kinds of writing, the time it takes to write is equivalent to the time it takes to wrap up a lesson or to do a few practice problems at the end of a lesson, with the added benefit students and teachers get from the insights of writing with elaboration.

Finally, since summary writing may hold the most promise for increasing achievement in mathematics, math teachers need to find the most effective way of teaching summaries to their students. In her article on the writing summaries, Hill (1991) identifies several types of writing and suggests that each type of summary writing “requires a different emphasis depending upon audience and purpose” (pp. 537-538), echoing the theories of Langer and Applebee (1987). Once again, teachers need to factor in the purpose and nature

of the content, as well as the nature and level of the learners in order to effectively use writing in the mathematics class.

SUGGESTIONS FOR FURTHER RESEARCH

There are many avenues of research to be explored in writing to learn in mathematics. Since summary writing was found to have the only significant effect on achievement in this study, further investigation into the most effective type of summary writing and instruction may yield more insight into the effectiveness of summaries. In addition, an area that was not addressed, due to time constraints, was the possible effect of the sequence of prompts on achievement. Does one type of prompt build on another? Another area of further research is the length of writing time. In this study, students wrote for only one school semester. What happens over time to student achievement when writing in these forms is a consistent part of the curriculum? As mentioned above, what happens when students engage in different types of writing that are selected according to the nature of the curriculum? Also, in this study writing instruction followed a particular protocol where students were led gradually from direct instruction to individual writing. What would happen if students were simply shown how to write in a particular ways? Many other factors, such as the length of time spent in writing instruction, different types of prompts, different types of elaboration, different grade levels, and different student populations, are also possible areas of research that may shed light on the intricate relationships between attitude, writing, and mathematics learning that were touched upon in this study and the many previous studies that informed this project. It is clear to this researcher that writing can be valuable tool for learning in mathematics by providing alternative, not add-on, activities to an already time-intensive curriculum.

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APPENDIX A

MINNESOTA MATHEMATICS ATTITUDE

INVENTORY

Minnesota Mathematics Attitude Inventory

	Strongly Agree	Agree	Disagree	Strongly Disagree
1. Mathematics is useful for the problems of every day life.	①	②	③	④
2. Mathematics is something which I enjoy very much.	①	②	③	④
3. I like the easy mathematics problems the best.	①	②	③	④
④ I don't do very well in mathematics.	①	②	③	④
5. My mathematics teacher shows little interest in the students.	①	②	③	④
6. Working mathematics problems is fun.	①	②	③	④
7. I feel at ease in a mathematics class.	①	②	③	④
8. I would like to do some outside reading in mathematics.	①	②	③	④
9. There is little need for mathematics in most jobs.	①	②	③	④
⑩ Mathematics is easy for me.	①	②	③	④
11. When I hear the word mathematics, I have a feeling of dislike.	①	②	③	④
12. Most people should study some mathematics.	①	②	③	④
13. I would like to spend less time in school doing mathematics.	①	②	③	④
14. Sometimes I read ahead in our mathematics book.	①	②	③	④
15. Mathematics is helpful in understanding today's world.	①	②	③	④
⑥ I usually understand what we are talking about in mathematics class.	①	②	③	④
17. My mathematics teacher makes mathematics interesting.	①	②	③	④
18. I don't like anything about mathematics.	①	②	③	④
⑨ No matter how hard I try, I cannot understand mathematics.	①	②	③	④
20. I feel tense when someone talks to me about mathematics.	①	②	③	④
21. My mathematics teacher presents material in a clear way.	①	②	③	④
⑫ I often think, "I can't do it," when a mathematics problem seems hard.	①	②	③	④
23. Mathematics is of great importance to a country's development.	①	②	③	④
24. It is important to know mathematics in order to get a good job.	①	②	③	④

	Strongly Agree	Agree	Disagree	Strongly Disagree
25. It doesn't disturb me to work mathematics problems.	①	②	③	④
26. I would like a job which doesn't use any mathematics.	①	②	③	④
27. My mathematics teacher knows when we are having trouble with our work.	①	②	③	④
28. I enjoy talking to other people about mathematics.	①	②	③	④
29. I like to play games that use numbers.	①	②	③	④
30. I am good at working mathematics problems.	①	②	③	④
31. My mathematics teacher doesn't seem to enjoy teaching mathematics.	①	②	③	④
32. Sometimes I work more mathematics problems than are assigned in class.	①	②	③	④
33. You can get along perfectly well in everyday life without mathematics.	①	②	③	④
34. Working with numbers upsets me.	①	②	③	④
35. I remember most of the things I learn in mathematics.	①	②	③	④
36. It makes me nervous to even think about doing mathematics.	①	②	③	④
37. I would rather be given the right answer to a mathematics problem than to work it out myself.	①	②	③	④
38. Most of the ideas in mathematics aren't very useful.	①	②	③	④
39. It scares me to have to take mathematics.	①	②	③	④
40. My mathematics teacher is willing to give us individual help.	①	②	③	④
41. The only reason I'm taking mathematics is because I have to.	①	②	③	④
42. It is important to me to understand the work I do in mathematics.	①	②	③	④
43. I have a good feeling toward mathematics.	①	②	③	④
44. My mathematics teacher knows a lot about mathematics.	①	②	③	④
45. Mathematics is more of a game than it is hard work.	①	②	③	④
46. My mathematics teacher doesn't like students to ask questions.	①	②	③	④
47. I have a real desire to learn mathematics.	①	②	③	④
48. If I don't see how to work a mathematics problem right away, I never get it.	①	②	③	④

APPENDIX B

UNIT PRE- AND POST-TESTS

What Do You Expect?
Pre-Test

1. Change these ratios to decimals and percents

a) $\frac{17}{100}$

b) $\frac{3}{4}$

c) $\frac{9}{10}$

d) $\frac{6}{25}$

2. You spin two spinners like the ones shown.

a) Make a chart or counting tree to find all the possible outcomes. Assume the spinners are cut into congruent (equal) areas.

b) List all the outcomes:

Find:

c) $P(C, 3)$

d) $P(A, 1)$

3. John and Karla take a survey of some students in their class about their pets. Here are their results:

Cat	
Dog	
Reptile	
Hamster	

Find:

a) $P(\text{Dog})$

b) $P(\text{Reptile})$

c) $P(\text{Cat or Hamster})$

d) Are these probabilities experimental or theoretical? Why?

e) According to the table, is owning a dog or hamster equally likely? Why or why not?

4. Raul and Maria have a bag containing:

6 blue cubes

3 white cubes

8 green cubes

Find:

a) $P(\text{red})$

b) $P(\text{white})$

c) $P(\text{green or blue})$

d) Are these probabilities experimental or theoretical? Why?

e) Are Raul and Maria equally likely to pull out a blue or white cube? Why or why not?

5. Write all answers in simplest form:

a) $\frac{7}{10} + \frac{1}{10} =$

b) $\frac{3}{4} + \frac{5}{8} =$

c) $\frac{2}{3} + \frac{1}{4} =$

d) $\frac{1}{7} \cdot \frac{1}{3} =$

6. A treasure was hidden randomly in one of the rooms shown on the 10x10 grid below.

Write your answer as a fraction decimal or percent for #5

a) What is the probability the treasure was hidden in the Living Room?

b) In what room was the treasure most likely to be hidden? Explain your reasoning.

c) If the game was played 100 times, how many times would you expect the treasure to be hidden in the Den?

7. A treasure was hidden in one of the rooms shown on the floor plan below.

Potato Chips	Snickers
Tortilla Chips	Skittles
	M&M's

- a. What is the probability that the treasure was hidden in a candy room? Explain your reasoning.
- b. What is the probability that the treasure was hidden in a chips room? Explain your reasoning.

8. Michael was analyzing a computer game. In his game, the arrangement of paths and forks leads into room A, room B, or room C. Michael made an area model to analyze the probability of ending in each room.

SHOW HOW YOU GET YOUR ANSWERS!

- a) What is the probability of ending in room A?

A	A	C
B		
A	B	

- b) What is the probability of ending in room B?

- c) What is the probability of ending in room C?

Variables and Patterns Pre-Test

1. One afternoon, Corey decided to run laps. He kept track of his progress in a table like the one at the left:

Time (Minutes)	Total laps around the track
0	0
10	5
20	10
30	14
40	19
50	22
60	24
70	24
80	32

a) Make a coordinate graph of the data given in the table.

b) How many laps did Cory run in all? How long did it take him?

c) During which time interval did Corey make the most progress? How do you know?

d) During which time interval did he make the least progress? How do you know?

2) The paths below show three possibilities of how Corey's speed may have changed during each 10 minutes period. Explain in writing what each connecting path would tell about his speed.

3) This graph shows how the price of stock in The Standley Math Club changed over the period of one day.

a) What are the two variables in this situation?

b) Make a table

c) During which time interval did the price rise the fastest? How do you know?

d) During which time interval did the price fall the fastest? How do you know?

e) How are the variables related (circle one): Explain how you know.

- A) As the hours increased, the stock price increased.
- B) The stock price increased only for the first few hours.
- C) As the hours increased, the stock price decreased.
- D) The stock price decreased for the first few hours, then increased.

4) The rule for the distance Elizabeth traveled in t hours was $d = 35t$.

a) Make a table showing how many miles Elizabeth would go in 0 to 8 hours.

b) Make a graph of your table. You may use a separate sheet of graph paper, if you like.

c) Would it make sense to connect the points in this situation? Why or why not?

d) How far would Elizabeth travel in 8 hours? How do you know?

e) How far did Elizabeth travel in $4\frac{1}{2}$ hours? How do you know?

f) How many hours did it take Elizabeth to travel 215 miles? How do you know?

g) Which representation, the graph or the table, did you use most to find your answers? Why?

5) You are going to work for your uncle over the summer for \$7.50 per hour.

a) Make a table of the amount you would get paid for 0 to 10 hours of work.

b) Write the rule for this situation in words and as an equation.

c) Graph the data from the table. You may use a separate sheet of graph paper if you'd like.

d) If your uncle would only pay you for a full hour's work, would it make sense to connect the points in this situation? Why or why not?

6. Anna is looking for a cellular phone company.

A-1 Cellular gave her a table of their prices for 1-10 months:

# of months	1	2	3	4	5	6	7	8	9	10
A-1's price	\$30	\$36	\$42	\$48	\$54	\$60	\$66	\$72	\$78	\$80

Fantastic Phones gave her a graph of their process:

- Fill in the table for Fantastic Phones and make a graph of A-1 Cellular.
- For how many months of use is A-1 Cellular the cheapest? How do you know?
- For how many months of use is Fantastic Phones the cheapest? How do you know?
- Which representation, the table or graph, helped you most in deciding which company is the cheapest?

7. The temperature in Alaska on Monday is 18 degrees. The weatherman predicts that the temperature will drop 22 degrees by Tuesday. If the weatherman is correct, what will the temperature be on Tuesday?

- a) 4 b) -40 c) -4 d) 40

8. Which equation models the situation above (x = Tuesday's temperature)? Explain your reasoning.

- a) $18 - 22 = x$ b) $22 - 18 = x$
c) $-18 + 22 = x$ d) $22 + 18 = x$

9. Which expression would show the number of degrees the temperature dropped if the temperature were to drop 5 degrees a day for 4 days. Explain your reasoning

- a) $-5 = 4$ b) $-4 + 5$ c) $-4(5)$ d) $4(-5)$

10. John got \$100 for his birthday. He owes three friends \$22.00 each. Choose the equation that shows how much money he will have left (x) after repaying his friends. Explain your reasoning.

- a) $100 + 3(-22) = x$ b) $100 - 3(-22)$
c) $x = 22(3) - 100$ d) $100x = 3(-22)$

11. How much money does John have left? Explain or show how you know.

Comparing and Scaling Pre-Test

Calculators allowed

SHOW WORK

1. Solve these proportions:

a) $\frac{2}{3} = \frac{x}{12}$

b) $\frac{4}{7} = \frac{6}{m}$

c) $\frac{n}{9} = \frac{5}{15}$

d) $\frac{8}{y} = \frac{10}{12}$

2. Write and solve proportions:

a) The scale on a map is 1 in:20 miles. If the actual distance from San Diego to San Juan Capistrano is 70 miles, how far apart will they be on the map?

b) Claudia finds she can mix 4 oz. of red paint with 9 oz. of yellow paint to make the perfect shade of orange for her project. How much yellow paint will Claudia need to mix with 10 oz. of red paint to make the same shade of orange?

3. Solve these equations by using inverse operations:

a) $n + 3.2 = 9.1$

b) $\frac{m}{5.9} = 8.2$

c) $9.3x = 8.5$

d) $y - 6.0 = 5.38$

4. The Seattle Seahawks won 9 games and lost 7.

a) What is the ratio of games lost to games won? _____

b) What is the ratio of games won to games played? _____

c) What percent of their games did they win? _____

d) Write a difference statement for this situation.

5. At Vons you can buy 6 packs of gum for \$2.52. At Save-On you can buy 9 packs of gum for \$3.96. Which is the better buy? How do you know? SHOW WORK

6. The 6 to 6 program wants to make grape juice from concentrate for snack time. They have four recipes shown below.

Recipe A	Recipe B
4 cups of concentrate	10 cups of concentrate
6 cups of water	14 cups of water
Recipe C	Recipe D
9 cups of concentrate	6 cups of concentrate
11 cups of water	10 cups of water

a) Which recipe will have the strongest grape flavor? How do you know?

b) Which recipe will have the weakest grape flavor? How do you know?

7. Write an equation relating to the cost (c) of the number of balloons (n) from each company.

- a) Party City 48 cents for 80 balloons _____
- b) Balloon Boys 90 cents for 180 balloons _____
- c) Bob's Balloons 84 cents for 105 balloons _____

8. A wildlife researcher caught and tagged 120 moose, then released them into the wild. From the following samples taken a few weeks later, predict the total number of moose in the wild. SHOW WORK.

Sample 1: $\frac{4\text{tagged}}{21\text{moose}}$

Sample 2: $\frac{8\text{tagged}}{24\text{moose}}$

Sample 3: $\frac{5\text{tagged}}{11\text{moose}}$

Sample 4: $\frac{3\text{tagged}}{10\text{moose}}$

ANSWER _____

9. Which city is most crowded, Dallas or Chicago?

- a) Dallas Population: 1,208,318 Area: 343 sq miles
- b) Chicago Population: 2,869,121 Area: 227 sq miles

ANSWER _____

Bonus Problem: Write a comparison statement about the population densities of Dallas and Chicago.

APPENDIX C
LISTING OF PROMPTS

Prompts

Group 1 Prompts

Summary prompts:

1. What are the important things to remember about from today's lesson on probability?

Give examples.

2. Give three important pieces of information about counting trees and write why they are important.

3. What are at least three important things to remember about changing fractions to percents?

4. What are least three important things to remember when changing a fraction to a percent by dividing?

5. Describe in detail at least two important things about area probabilities.

6. What are at least two important things to remember about adding fractions with like denominators?

7. What are at least two important things to remember about experimental and theoretical probabilities and how you can tell them apart?

8. Choose a problem and a way to find the common denominator. Explain how to do the problem your way. [This prompt was not supposed to given to this group]

9. Summarize what will be on the test on Tuesday. Write about any important ideas we have discussed over the past 5 weeks. Give examples or say why they are important.

[Students had just finished a review and had access to their reviews, notebooks, and blue books]

10. Write at least three important things to remember about multiplying fractions.

Metacognitive (self-monitoring) prompts:

1. What do you know about graphing? What are you still unsure about?
2. What do you know about reading graphs? What are you still unsure about?
3. What do you understand about making a table [from a graph]? What do you not understand?
4. What do you understand about making a table [from a rule]? What do you not understand?
5. What do you know about comparing graphs? What are you still not sure about?
6. What do you understand about finding equations from tables? What are you unsure about?
7. What do you understand about dependent and independent variables? What do you not understand?
8. What do you understand about making a rule [equation] from a table? What do you not understand?
9. What do you understand about speed [time and distance] graphs? What questions/problems do you still have?
10. Write down all the things you understand and do not understand about the quiz and/or today's work [review].

Procedural Prompts:

1) Describe how to write the two types of comparison statements below for the following date: The Chargers won 14 and lost 2. Total games: 16

a) A difference statement

b) A part to whole ratio statement

2) How do you change a ratio to a percent? [Students were given the following data to use an example]

Last year, the Padres won 88 games and lost 75. What percent of their games did they win?

3) How do you find a percent with parts? [Students were given the following data to use an example]

Sport	Boys	Girls
Basketball	3	9
Track and Field	3	4
Total surveyed: 27		

4) How do you find which item is the best buy? [Students were given the following data to use an example]

6 oz. of lotion for \$4.35 or 8 oz. of lotion for \$6.80

5) Describe how to solve the following equations:

a) $x + 3.2 = 5.19$

b) $n - 3.87 = 0.25$

6) How do you solve a proportion like $\frac{n}{8} = \frac{9}{12}$?

7) How do you write and solve a proportion? You may write your own example or use the following:

Fred can eat 3 bicycles in 2 hours. How long would it take him to eat 10 bicycles?

8) Describe step by step how to: a) find the unit rate and b) write an equation using the unit rate for the following situation:

Daniel can eat 19 pizzas in 4 hours (you may make up your own problem, if you'd like)

9) How do you find a population density? [students were given the following data to use as their example]

The population of San Diego is 1,255,540 and San Diego covers an area of 324 mi².

10) Choose at least one problem from the review and describe in detail how to solve it. Include examples and why you did what you did.

Group 2 Prompts

Procedural prompts:

1. In the problem below, describe step-by-step how to answer the questions.

Luz had a bag with 5 blue marbles, 8 green marbles, and 2 black marbles.

a) What are the outcomes in this problem?

b) Find: $P(\text{blue})$

$P(\text{black or green})$

$P(\text{not green})$

2. Describe step-by-step how to use a counting tree to find the outcomes of two independent events.

3. Describe how to change a fraction to a percent.

4. Describe how to change a fraction to a percent by dividing.

5. Describe how to find the probability of hitting Area A in the diagram below. Assume area $A = \text{area B} + \text{Area C}$ and that $\text{area 1} = \text{area 2} = \text{area 3}$. ["Hitting Area A" was defined in the curriculum in the context of a video game, where a computer randomly placed a treasure in a room]

6. Describe how to add fractions with like denominators.
7. Describe how to tell whether a probability is experimental or theoretical.
8. Describe how to find a common denominator using one of the three ways described in class.
9. Explain how to change a fraction into a decimal and why you took the steps you took.
10. Describe how to multiply $\frac{5}{6} \bullet \frac{3}{10}$.

Summary Prompts:

1. Write about at least three things to remember about graphs.
2. Write about at least three things to remember about reading graphs.
3. Write about at least three important things to remember about making tables from graphs.

4. Write about at least three important things to remember when making a table from an equation.
5. What are at least three important things to remember when comparing graphs.
6. What are at least three important things to remember about finding equations from tables?
7. What are the important things to remember about dependent and independent variables?
8. What are the important things to remember about making tables from rules (equations), such as $d=35t$?
9. What are the important things to remember about speed [time/distance] graphs?
10. Summarize what will be on the test on Thursday.

Metacognitive (self-monitoring) Prompts:

- 1) What do you know about comparison statements? What are you still not sure about?
- 2) What do you know about changing ratios to percent with a calculator? What are you still not sure about?
- 3) What do you know about finding a percent with parts? What are you still unsure about?
- 4) What do you understand about finding the best buy? What do you not understand?
- 5) What do you know about solving each of the types of equations below? What are you still unsure about?

a) $x + 3.2 = 5.19$ b) $n - 3.87 = 0.25$ c) $8.6y = 12.35$ d) $\frac{m}{0.8} = 9.33$

6) What do you know about solving problems like $\frac{4}{15} = \frac{n}{75}$?

7) What do you know about writing and solving proportions? What do you not understand?
[Students were given the following example to use]

Fred can eat 3 bags of Hot Cheetos in 2 hours. How long would it take him to eat 10 bags of Hot Cheetos?

8) What do you know about finding unit rates and writing an equation using the unit rate?
What are you still unsure about? [Students were given the following example to use]

Daniel can eat 19 pizzas in 4 hours.

9) What do you know about finding population density? What do you not understand?

10) Look over your stations (review) work from the last 2 days. Write about what you understand and about what you still need to know.

Group 3 Prompts

Metacognitive (self-monitoring) Prompts:

1. What do you know about probability so far? What do you still need to know?
2. What do you know about making counting trees and what are you still unsure about?
3. What do you know about changing a fraction to a percent? Use examples to support your statements. What do you still not understand?
4. What do you understand about changing a fraction to a percent by dividing? What do you still not understand?
5. What do you understand about finding area probabilities? What do you not understand?
6. What do you understand about adding fractions with like denominators? What do you not understand?
7. What do you know about deciding whether a probability is experimental or theoretical?
8. Explain what you know about common denominators. Give examples. What questions do you still have?
9. Look over the “Stations” activity and the review we just did. What parts of these activities do you understand? Be specific. What questions do you still have?
10. What do you understand about multiplying fractions? What do you not understand?

Procedural Prompts:

1. Describe how to make a graph from a table.

2. Describe step by step how to read a graph like those we've been reading in class.
3. Describe how to make a table from a graph.
4. Describe how to make a table from an equation.
5. Describe how to compare two graphs on the same coordinate plane.
6. Describe how to find an equation from a table.
7. How do you decide whether a variable is "dependent" or "independent"?
8. Describe how to make a table from each of the four equations below. Make sure you give an example of each rule.
 - a) $d=20t$
 - b) $y=4x + 1$
 - c) $n=20-m$
 - d) $y=25-3x$
- 9) How do you analyze a "speed" graph?
- 10) Choose a problem on the test and describe, with details, explanation, and examples, how to do it.

Summary Prompts:

- 1) What are at least three important things to remember about writing comparison statements? [students were given the following data to use as their example]

The Chargers won 14 and lost 2. They played a total of 16 games.

- 2) What are at least 3 important things to remember about changing ratios to percents with a calculator? [students were given the following data to use as their example]

Last year, the Padres won 88 games and lost 75. What percent of their games did they win?

- 3) What are at least three important things to remember about finding percents with parts?
[students were given the following data to use as their example]

Mix A had 2C of orange concentrate and 3C of water. What percent of the mixture was orange concentrate?

- 4) What are the most important things to remember about finding the best buy?
[students were given the following data to use as their example]

24 pencils for \$3.88 or 40 pencils for \$5.70

- 5) What are the most important things to remember about solving equations like:

a) $x + 3.2 = 5.19$ b) $n - 3.87 = 0.25$ c) $8.6y = 12.35$ d) $\frac{m}{0.8} = 9.33$

- 6) What are the three most important things to remember about solving proportions?

- 7) What are the most important things to remember about writing and solving proportions?
[students were given the following situation to use as their example]

Fred can eat 3 pizzas in 2 hours. How long would it take him to eat 10 pizzas?

- 8) What are the most important things to remember about finding and equation using a unit rate?

- 9) What are the most important things to remember about finding a population density?
[students were given the following data to use as their example]

The population of San Diego is 1,255,540 and San Diego covers an area of 324 mi^2 .

10) Look at the stations work [review] from the last 2 days. What are the most important things to remember about the unit? Include examples and whys.

Or

Summarize what the unit was about. Include examples and whys.

What Do You Expect Unit Writing Evaluation - Summary

For the past 5 weeks you have been writing summaries of the lessons in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written summaries about probability, counting trees, two ways of changing fractions to percents, adding fractions, and using area models to find probabilities.

Has summary writing, writing about the important points in a lesson, helped you think about, learn, and/or remember the mathematics in probability, fractions, decimals, and percents? Why or why not?

What Do You Expect Unit Writing Evaluation - Procedures

For the past 5 weeks you have been writing procedures (how to) in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written procedures about probability, counting trees, two ways of changing fractions to percents, adding fractions, and using area models to find probabilities.

Has writing procedures, or how to do mathematical problems, helped you think about, learn, and/or remember the mathematics in probability, fractions, decimals, and percents? Why or why not?

What Do You Expect Unit Writing Evaluation – Self-Monitoring

For the past 5 weeks you have been self-monitoring your learning through writing in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written about probability, counting trees, two ways of changing fractions to percents, adding fractions, and using area models to find probabilities.

Has writing about what you do and do not know about the mathematics we've been doing helped you think about, learn, and/or remember the mathematics of probability, fractions, decimals, and percents? Why or why not?

Unit 2 Writing Evaluation - Summary

For the past 5 weeks you have been writing summaries of the lessons (writing down the important points in a lesson) in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written summaries about making coordinate graphs, reading graphs, making tables from graphs and equations, comparing graphs, finding equations from tables, and speed graphs.

Has summary writing, writing about the important points in a lesson, helped you think about, learn, and/or remember the mathematics of making tables, graphs, and equations? Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on so far this year better, writing how to do a problem (procedural writing) or summarizing the important points about a lesson? Why?

Unit 2 Writing Evaluation - Procedures

For the past 5 weeks you have been writing procedures, or how to do problems, in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written procedures about making coordinate graphs, reading graphs, making tables from graphs and equations, comparing graphs, finding equations from tables, and speed graphs.

Has procedural writing, explaining how to do different types of problems in writing, helped you think about, learn, and/or remember the mathematics of making tables, graphs, and equations?

Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on so far this year better, self-monitoring (writing about what you do and don't know about a mathematical problem or idea), or how to do a problem(procedural writing)? Why?

Unit 2 Writing Evaluation – Self-Monitoring

For the past 5 weeks you have been self-monitoring your understanding (writing about what you do and do not understand about a lesson, skill, or concept) in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written self-monitoring reflections about making coordinate graphs, reading graphs, making tables from graphs and equations, comparing graphs, finding equations from tables, and speed graphs.

Has self-monitoring, writing about what you do and do not understand about a problem, helped you think about, learn, and/or remember the mathematics of making tables, graphs, and equations?

Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on so far this year better, summaries (writing about the important points in a lesson), or self-monitoring (writing about what you do and do not understand about a lesson, skill, or concept)? Why?

Unit 3 Writing Evaluation - Summary

For the past 5 weeks you have been writing summaries of the lessons (writing down the important points in a lesson) in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written summaries about writing and solving proportions, finding better buys, finding percents from ratios, population densities, writing equations, and solving equations.

Has summary writing, writing about the important points in a lesson, helped you think about, learn, and/or remember the mathematics of writing and solving proportions and equations?

Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on this year better, self-monitoring (writing about what you do and don't know about a mathematical problem or idea), summary writing (writing about the important ideas of a lesson or concept, or procedural writing (how to do a problem)? Why?

Unit 3 Writing Evaluation - Procedures

For the past 5 weeks you have been writing procedures, or explaining how to do problems, in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written procedures about writing and solving proportions, finding better buys, finding percents from ratios, population densities, writing equations, and solving equations.

Has procedural writing, explaining how to do different types of problems in writing, helped you think about, learn, and/or remember the mathematics of writing and solving proportions and equations?

Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on this year better, self-monitoring (writing about what you do and don't know about a mathematical problem or idea), summary writing (writing about the important ideas of a lesson or concept, or procedural writing (how to do a problem)? Why?

Unit 3 Writing Evaluation – Self-Monitoring

For the past 5 weeks you have been self-monitoring your understanding (writing about what you do and do not understand about a lesson, skill, or concept) in your blue book to help you think about, learn, and remember the mathematics we have been studying. We have written self-monitoring reflections about writing and solving proportions, finding better buys, finding percents from ratios, population densities, writing equations, and solving equations.

Has self-monitoring, writing about what you do and do not understand about a problem, skill, or concept, helped you think about, learn, and/or remember the mathematics of writing and solving proportions and equations?
Why or why not?

Which kind of writing has helped you understand and/or remember the math we've been working on this year better, self-monitoring (writing about what you do and don't know about a mathematical problem or idea), summary writing (writing about the important ideas of a lesson or concept, or procedural writing (how to do a problem)? Why?